FX Derivatives: Model and Product Trends

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Agenda

1. Vanna-Volga Approaches
   - Model History
   - Wystup/Traders' Rule of Thumb
   - Castagna/Mercurio
   - Design Issues
   - Consistency Issues
2. Stochastic-Local-Volatility
   - Model History
   - Example: Tremor (Murex)
   - Example: Jäckel/Kahl's Hyp Hyp Hooray
   - Example: Andersen-Hutchings (PricingPartners)
   - Example: Silvano
   - Example: Pagliarani & Pascucci
   - Example: Bloomberg
3. FX Derivatives Product/Platform Trends
   - Target Forwards
   - FX Derivatives in Digital Casinos
   - FX-Linked Swaps in City-Treasury Departments and Family Offices
   - FX CFDs on Brokerage Platforms

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Other refs [Shkolnikov, 2009], [Janek, 2011]

Wystup/Traders' Rule of Thumb 2003

[Wystup, 2003], [Wystup, 2006]: compute the cost of the *overhedge* of risk reversals (RR) and butterflies (BF) to hedge vanna and volga of an option EXO.

$$\text{VV-value} = \text{TV} + p[\text{cost of vanna} + \text{cost of volga}] \quad (1)$$

with

$$\begin{aligned} \text{cost of vanna} &= \frac{vannaEXO}{vannaRR} \times \text{OH RR} && (2)\\ \text{cost of volga} &= \frac{volgaEXO}{volgaBF} \times \text{OH BF} && (3)\\ p &= \text{no-touch probability or modifications} && (4)\\ \text{OH} &= \text{overhedge} = \text{market price} - \text{TV} && (5) \end{aligned}$$

Castagna/Mercurio 2007

[Castagna and Mercurio, 2007], [Castagna and Mercurio, 2006]: portfolio of three calls hedging an option risk up to second order (in particular the vanna and volga of an option.

$$c(K, \sigma_K) = c(K, \sigma_{BS}) + \sum_{i=1}^{3} x_i(K)[c(K_i, \sigma_i) - c(K_i, \sigma_{BS})] \tag{6}$$

with

$$\begin{aligned} x_1(K) &= \frac{\frac{\partial c(K,\sigma_{BS})}{\partial \sigma}}{\frac{\partial c(K_1,\sigma_{BS})}{\partial \sigma}} \frac{\ln \frac{K_2}{K} \ln \frac{K_3}{K}}{\ln \frac{K_2}{K_1} \ln \frac{K_3}{K_1}} \\ x_2(K) &= \frac{\frac{\partial c(K,\sigma_{BS})}{\partial \sigma}}{\frac{\partial c(K_2,\sigma_{BS})}{\partial \sigma}} \frac{\ln \frac{K}{K_1} \ln \frac{K_3}{K}}{\ln \frac{K_2}{K_1} \ln \frac{K_3}{K_2}} \\ x_3(K) &= \frac{\frac{\partial c(K,\sigma_{BS})}{\partial \sigma}}{\frac{\partial c(K_3,\sigma_{BS})}{\partial \sigma}} \frac{\ln \frac{K}{K_1} \ln \frac{K}{K_2}}{\ln \frac{K_3}{K_1} \ln \frac{K_3}{K_2}}. \end{aligned} \tag{7}$$

Vanna-Volga Design Issues

- Hedge with BF and RR vs Hedge with 3 Vanillas, and if vanillas then which ones?

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- Not clear how to translate to Double-Touch products
- Which volatility to use for the touch probability: ATM, average of ATM and barrier vol, derived from equilibrium condition NTvv=NTbs+NTvv*...

Comparison: Heston-Local-VV EUR-USD OT Mustache

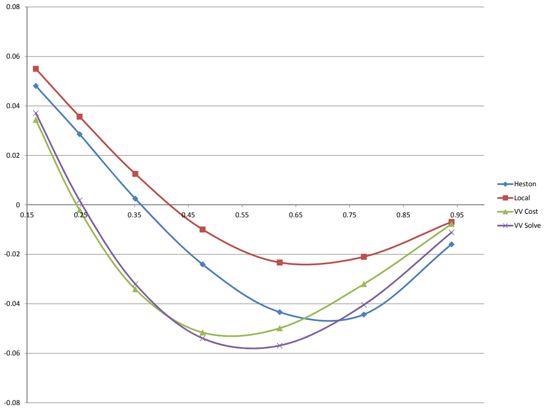


Figure: Model Comparison 6-M-EUR-USD-OT down paying USD: Market data of July 11 2012

Comparison: Heston-Local-VV EUR-USD DNT Mustache

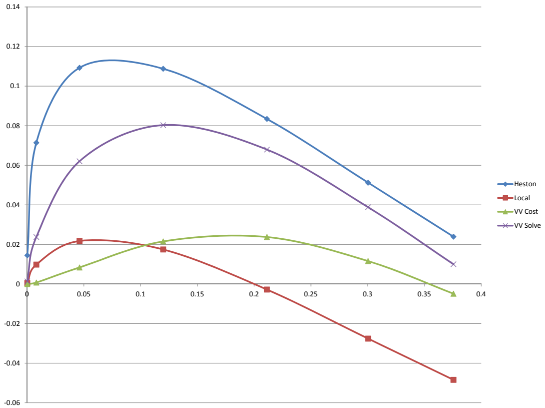


Figure: Model Comparison 6-M-EUR-USD-DNT paying USD: Market data of July 11 2012

Comparison: Heston-Local-VV EUR-USD Down-Out-Call Mustache

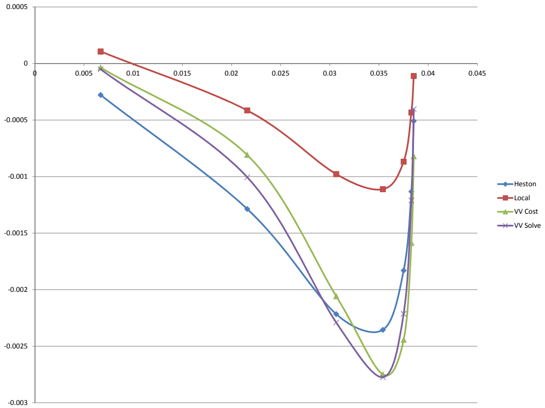


Figure: Model Comparison 6-M-EUR-USD down-out-call: Market data of July 11 2012

Vanna-Volga Consistency Issues

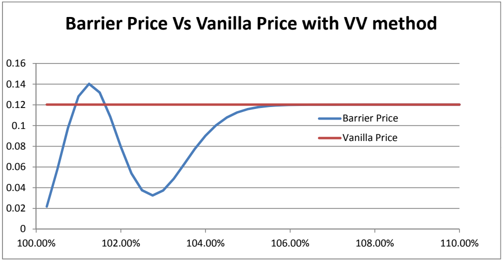
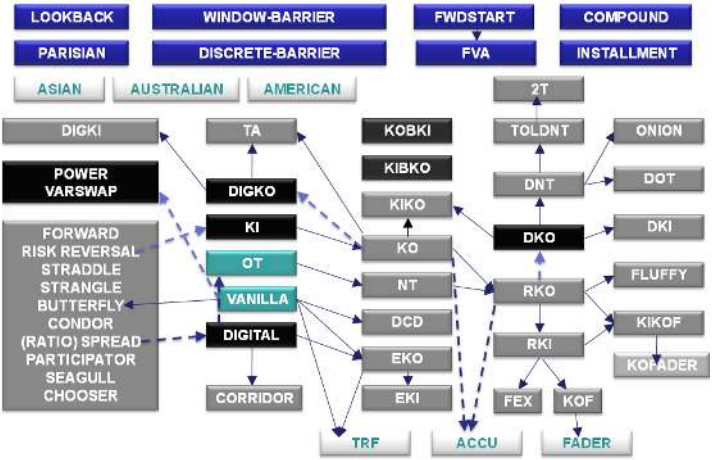


Figure: Convergence of a RKO EUR call CHF put to vanilla, strike 1.0809, 60 days, Market data of April 11 2012: Spot ref 1.20105, 2M EUR rate 0.055%, 2M-Forward -5.65, 10D BF 4.10, 25D BF 1.4755, ATM 3.00, 25D RR -0.7010, 10D RR -1.70.

Vanna-Volga Consistency Issues



Stochastic-Local Volatility Model History

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$$\mathrm{d}S/S = r\,\mathrm{d}t + \sigma(1 + \alpha(S - S_0) + \beta(S - S_0)^2)\mathrm{d}W$$
$$\mathrm{d}\sigma = \kappa(\theta - \sigma)dt + \epsilon\,\sigma\mathrm{d}Z$$

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- In [Lipton, 2002] Lipton describes an LSV model using a mean reverting process for the variance (like in the Heston model) and a general local volatility function $\sigma_L(t,S)$ as multiplicator for the stochastic volatility in the equation for the spot process.

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- Several LSV models using different choices of local volatility functions in the equation for the spot or forward process combined with a stochastic process for the volatility, the variance, or a function of variance, have been considered and implemented in the following years in the industry. (See for one example [de Backer, 2006].)

What Murex calls "Tremor"

Model [1] the forward as ([Murex, 2008b], [Murex, 2008a])

$$
\begin{aligned}
\mathrm{d}f_t^T &= \sqrt{v_t}\cdot g(f_t^T)\mathrm{d}W_t^{(1)}, && (8)\\
\mathrm{d}v_t &= \kappa(\theta - v_t)\mathrm{d}t + \eta\sqrt{v_t}\mathrm{d}W_t^{(2)} && (9)\\
g(f) &= \min(a + bf + cf^2, \mathrm{cap}\cdot f) && (10)\\
& \mathrm{Cov}(\mathrm{d}W_t^{(1)}, \mathrm{d}W_t^{(2)}) = \rho\mathrm{d}t && (11)
\end{aligned}
$$

[1] The name "Tremor" originates from Pascal Tremoureux, a senior member of the quant team at Murex

What Murex calls "Tremor"

- f_t^T forward to maturity T seen from t
- v_t variance process
- v_0 initial instantaneous variance
- θ long-term variance set to v_0
- $\kappa > 0$ rate of mean-reversion
- ρ instantaneous correlation between f_t^T and v_t
- η volatility of variance (vol of vol)
- a, b, c parameters of g the *local volatility function*
- cap: 500%

Jäckel/Kahl's Hyp Hyp Hooray

LSV model [2] introduced by Jäckel and Kahl in 2007 ([Jäckel and Kahl, 2010]), within equity options context, both of hyperbolic type in the parametric local vol and stochastic vol:

$$\begin{aligned}
\mathrm{d}x &= \sigma_0 \cdot f(x) \cdot g(y) \mathrm{d}W_t^{(1)}, && (12)\\
\mathrm{d}y &= -\kappa y \mathrm{d}t + \alpha\sqrt{2\kappa}\mathrm{d}W_t^{(2)} && (13)\\
f(x) &= \left[(1-\beta+\beta^2)\cdot x + (\beta-1)\cdot\left(\sqrt{x^2+\beta^2(1-x^2)}-\beta\right)\right]/\beta && (14)\\
g(y) &= y + \sqrt{y^2+1} && (15)\\
& \mathrm{C}ov(\mathrm{d}W_t^{(1)}, \mathrm{d}W_t^{(2)}) = \rho \mathrm{d}t && (16)
\end{aligned}$$

WLOG $\beta > 0$, $g(0) = 1$, $x(0) = 1$, $f(1) = 1$.

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Pros:

- analytical approximation for implied volatilities

[2] SD uses a version of this model [Vozovoi and Klein, 2011]

SuperDerivatives Screen Shot

Figure: SuperDerivatives Screen Shot with SLV mid price SuperDerivative.com

Contained in PricingPartners: "Andersen-Hutchings"

[Andersen and Hutchings, 2008], a generalization of [Piterbarg, 2005], consider a normalized asset

$$\mathrm{d}X_t = \lambda(t)\sqrt{V_t}\left(1 - b(t) + b(t)X_t + \frac{1}{2}c(t)(X_t - 1)^2\right)\mathrm{d}W_t^{(1)}, \quad (17)$$

$$\mathrm{d}V_t = \kappa(1 - V_t)\mathrm{d}t + \eta(t)\sqrt{V_t}\mathrm{d}W_t^{(2)}, V_0 = 1, X_0 = 1 \quad (18)$$

$$\mathrm{C}ov(\mathrm{d}W_t^{(1)}, \mathrm{d}W_t^{(2)}) = \rho(t)\mathrm{d}t \quad (19)$$

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The model parameters are:

- $b(t)$ is the linear skew parameter, being a time-dependent function.
- $c(t)$ is the quadratic skew parameter, being a time-dependent function.
- $\lambda(t)$ is a time-dependent function.
- κ is the mean reversion speed of volatility.
- $\eta(t)$ is the volatility of variance, being a time-dependent function

"Silvano"

$$
\begin{aligned}
\mathrm{d}S_t &= (r_t^d - r_t^f)S_t\mathrm{d}t + \sigma_t S_t \mathrm{d}W_t^{(1)}, &(20)\\
\mathrm{d}V_t &= \eta_t(\ln z_t\theta_t - \ln V_t)V_t\mathrm{d}t + \Sigma_t\mathrm{d}W_t^{(2)}, &(21)\\
L_t &= f(S_t, \Sigma_t, \rho_t, t), &(22)\\
\sigma_t &= \min[\text{cap}, z_t(\omega_t V_t + (1-\omega_t)L_t)], &(23)\\
\mathrm{Cov}[\mathrm{d}W_t^{(1)}, \mathrm{d}W_t^{(2)}] &= \rho_t \mathrm{d}t, &(24)
\end{aligned}
$$

where

"Silvano"

S_t	:	spot price	(25)
r_t^d	:	domestic rate	(26)
r_t^f	:	foreign rate	(27)
σ_t	:	volatility	(28)
V_t	:	stochastic volatility	(29)
η_t	:	mean reversion speed of stochastic volatility	(30)
θ_t	:	equilibrium level of stochastic volatility	(31)
Σ_t	:	volatility of volatility	(32)
z_t	:	volatility drift factor	(33)
L_t	:	local volatility	(34)
ω_t	:	stochastic volatility weight	(35)
ρ_t	:	spot-volatility correlation	(36)
cap	:	volatility cap, currently set at 1000%	(37)
f	:	local volatility function	(38)

Pagliarani & Pascucci: SLV with Jumps

Risk-neutral dynamics for the log-price of the underlying asset [Pascucci and Pagliarani, 2012]

$$
\begin{aligned}
\mathrm{d}X_t &= \left(\bar{r} - \mu_1 - \frac{\sigma^2(t, X_{t-})v_t}{2}\right) S_t \mathrm{d}t + \sigma(t, X_{t-})\sqrt{v_t}\mathrm{d}W_t^{(1)} + \mathrm{d}Z_t, \quad (39) \\
\mathrm{d}v_t &= k(\theta - v_t)V_t \mathrm{d}t + \eta\sqrt{v_t}\left(\rho \mathrm{d}W_t^{(1)} + \sqrt{1-\rho^2}\mathrm{d}W_t^{(2)}\right), \quad (40)
\end{aligned}
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$$\mathrm{d}v_t = k(\theta - v_t)V_t\mathrm{d}t + \eta\sqrt{v_t}\left(\rho \mathrm{d}W_t^{(1)} + \sqrt{1-\rho^2}\mathrm{d}W_t^{(2)}\right), \quad (40)$$

- $\bar{r} = r_d - r_f - \int (e^y - 1 - y\boldsymbol{I}_{\{-1<y<+1\}})\nu(\mathrm{d}y)$
- Z pure-jump Lévy process independent of the W, with triplet $(\mu_1, 0, \nu)$
- Model is characterized by the local volatility function σ, the variance parameters: initial variance v_0, speed of mean reversion k, long-term variance θ, vol-of-vol η and correlation ρ; and the Lévy measure ν.

Analytical approximation of the characteristic function.

Bloomgerg's SVL Model [Tataru et al., 2012]

$$\frac{\mathrm{d}S_t}{S_t} = (r_t^d - r_t^f)\mathrm{d}t + L(S_t, t)V_t\mathrm{d}W_t^{(1)}, \quad (41)$$

$$\mathrm{d}V_t = \kappa(\theta - V_t)\mathrm{d}t + \xi V_t \mathrm{d}W_t^{(2)}, \quad (42)$$

$$\mathrm{Cov}[\mathrm{d}W_t^{(1)}, \mathrm{d}W_t^{(2)}] = \rho_t \mathrm{d}t, \quad (43)$$

Bloomgerg's SVL Model [Tataru et al., 2012]

$$\frac{dS_t}{S_t} = (r_t^d - r_t^f)dt + L(S_t, t)V_t dW_t^{(1)}, \quad (41)$$

$$dV_t = \kappa(\theta - V_t)dt + \xi V_t dW_t^{(2)}, \quad (42)$$

$$\mathrm{Cov}[dW_t^{(1)}, dW_t^{(2)}] = \rho_t dt, \quad (43)$$

where the local volatility L and stochastic volatility V are mixed using $0 \leq \lambda \leq 1$ for

$$\xi = \lambda \xi_{\max}, \qquad \rho = \lambda \rho_{\max}. \quad (44)$$

Bloomberg Screen Shot: Local Stochastic Vol Parameters

FX Derivatives Product Trends: Target Forwards

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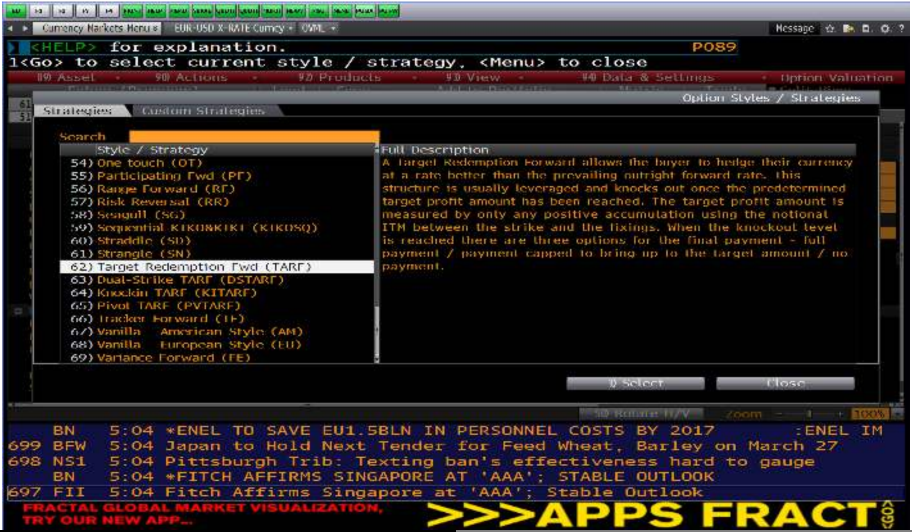
4. Pricing/Platforms

Bloomberg Screen Shot: Target Redemption Forward (TARF)



Bloomberg Screen Shot: OVML Pivot Target Forward

Pivot Target Forward P&L Scenarios

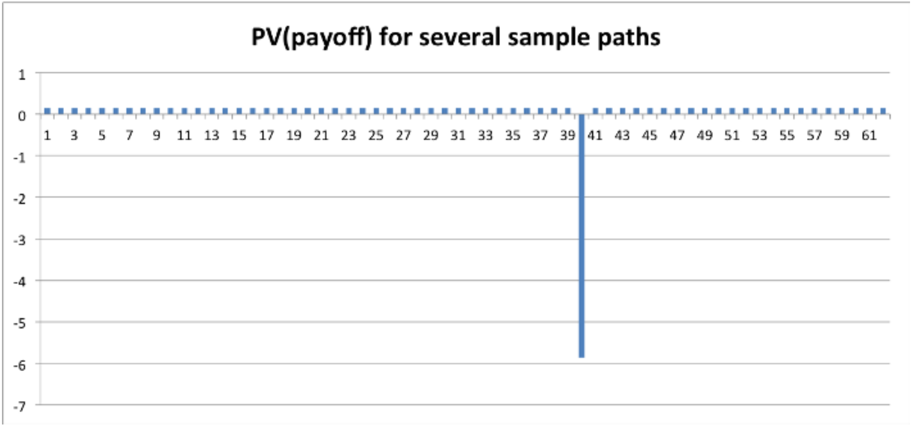


Figure: P&L Scenarios Pivot Target Forward

Pivot Target Forward Payoff and Psychology

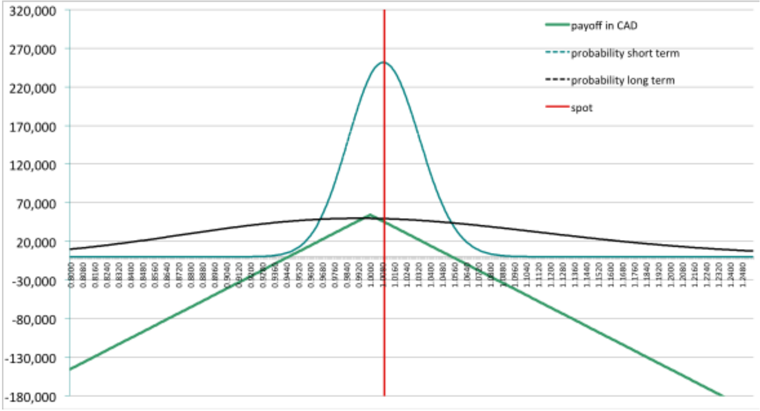


Figure: Pivot Target Forward Payoff and Psychology

FX Derivatives in Digital Casinos

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Currency Related Swap

6M-EURIBOR + spread

$\longrightarrow$

bank | semiannual payments | treasurer

$\longleftarrow$

$$\text{6M-EURIBOR} + \frac{(\text{Strike}-\text{EUR-CHF-Fixing})^{+}}{\text{EUR-CHF-Fixing}} \times 100\%$$

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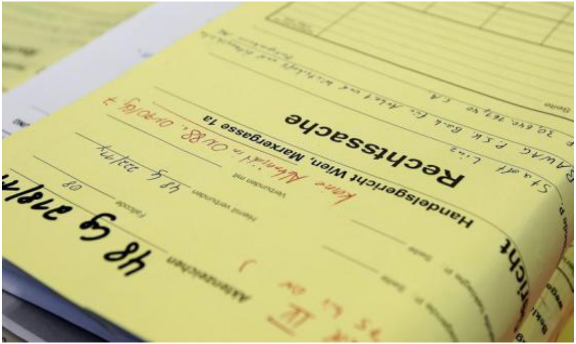
Example: City of Linz ./. BAWAG



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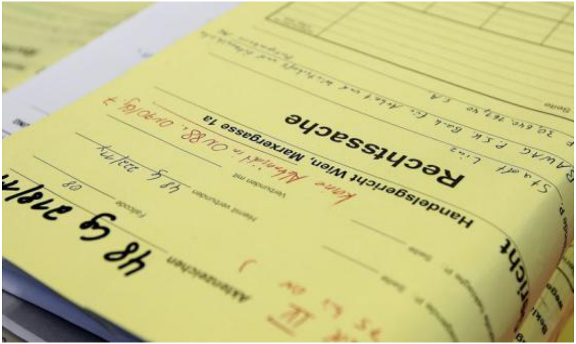

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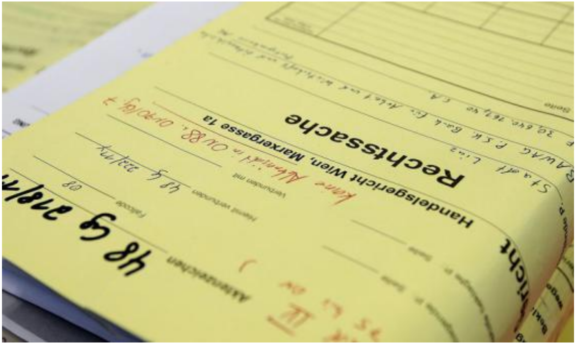

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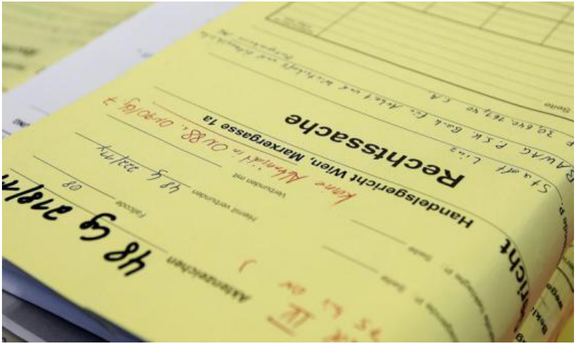

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- CHF-loans done for the purpose of reducing interest rate payments require active position management.

FX CFDs on Brokerage Platforms

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Comparison: DNT Mustache

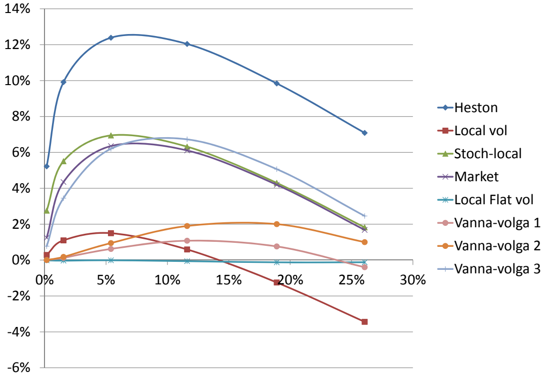

Figure: Model Comparison 6-M-EUR-USD-DNT paying USD: Spot ref 1.3068, 6M USD rate 0.11%, 6M-Forward 14.40, 10D BF 1.85, 25D BF0.45, ATM 11.99, 25D RR

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4. Vanna-volga resurrects in digital casinos and private banking.

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