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Working Paper

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CPQF Working Paper Series, No. 25

Provided in Cooperation with:
Frankfurt School of Finance and Management

Suggested Citation: Beyna, Ingo; Wystup, Uwe (2010) : On the calibration of the Cheyette interest rate model, CPQF Working Paper Series, No. 25

This Version is available at:
http://hdl.handle.net/10419/43669

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CPQF Working Paper Series

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June 2010

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On the Calibration of the Cheyette Interest Rate Model

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June 28, 2010

Abstract

We investigate the robustness of existing methods to calibrate the Cheyette interest rate model to at-the-money swaption, caps and floors. Existing algorithms may fail, because they suffer from numerical instability of derivatives. Therefore, we apply derivative-free techniques and find that they stabilize the calibration. Furthermore, we identify auspicious volatility parametrizations determining the Cheyette model. In combination with the established calibration techniques the results imply an accurate market reproduction and stay robust against changes in the initial values. In contrast to existing approaches that use approximations, we apply exact semi-close-form pricing formulas.

Keywords: Cheyette Model, Calibration, Optimization without derivatives, Genetic Optimization

*We would like to thank Prof. Dr. P. Roßbach (Frankfurt School of Finance & Management), Prof. Dr. H. Mittelmann (Arizona State University) and Prof. Dr. W. Schmidt (Frankfurt School of Finance & Management) for their support and advice on this paper.

Contents

1 Introduction

In 1992, D. Heath, R. Jarrow and A. Morton (HJM) [Mor92] have standardized a valuation approach on the basis of mainly two assumptions: the first one postulates, that it is not possible to gain riskless profit (No-arbitrage condition), and the second one assumes the completeness of the financial market. The HJM model, or strictly speaking the HJM framework, is a general model environment and incorporates many previously developed models like the Vasicek model (1977) [Vas77] or the Hull-White model (1990) [Whi90]. The general setting mainly suffers from two disadvantages: first of all the difficulty to apply the model in market practice and second, the extensive computational complexity caused by the high-dimensional stochastic process of the underlying. The first disadvantage was improved by the development of the LIBOR Market Model (1997) introduced by [Mus97], [Jam97] and [San97], which combines the general risk-neutral yield curve model with market standards. The second disadvantage can be improved by restricting the general HJM model to a subset of models with a similar specification of the volatility structure. The resulting system of Stochastic Differential Equations (SDE) describing the yield curve dynamic breaks down from a high-dimensional process into a low-dimensional structure of Markovian processes. Furthermore, the dependence on the current state of the process allows the valuation by a certain Partial Differential Equation (PDE). This approach was developed by O. Cheyette in 1994 [Che94].

The framework proposed by Cheyette for modeling interest rates leaves the user with a wide choice of alternatives. The model is completely determined by the selection of (separable) volatility functions and given a parametrization, one might end up in known models like Ho-Lee (constant volatility) or Hull-White (exponential volatility structure). Each model bears different advantages and the selection of a model depends on its application. In order to use one model in practice it needs to be calibrated to liquidly traded interest rate derivatives. The aim of the calibration is minimizing the differences in prices between the market quotes and the model implied prices. The minimization is performed with respect to the free parameters of the model in case of the Cheyette framework to the coefficients of the volatility parametrization. Alternatively, the calibration can be implemented to minimize the differences in implied (Black-Scholes) volatility. The comparison of the volatilities is more standardized, because it is independent of the notional and the maturity. Furthermore, the quotation in the market of prices and implied volatility coincides for plain products, thus the calibration problem stays unchanged.

The calibration of an interest rate model is one of the most important steps for pricing exotic products. Surprisingly, very little literature is known dealing with the applicability of optimization algorithms. The special structure of the Cheyette model influences the performance of the minimization methods. The pricing formulas are given semi-explicitly [Hen03] and their derivations are not given in closed-form. One should thus apply derivative-free minimization algorithms like the Downhill Simplex algorithm or Genetic Optimization techniques. In our work we have applied several methods to the calibration problem and in this paper we discuss their behavior in detail. The results are quite different for the analyzed techniques and the choice of an adequate optimization algorithm plays an important role. As the calibration problem seeks a global minimum, we have incorporated some stochastic algorithms which do not guarantee the detection of the global minimum, but reach it with high probability. Furthermore, its application to the calibration of interest rate models is not well explored yet and we give a starting point in this paper.

2 Literature Review

The calibration of interest rate models to plain derivatives can be formulated theoretically quite easy as a minimization of the differences in prices between the model and the market quotes. Taking the specific pricing formulas into account, the characteristics of the optimization problem appear. Brigo and Mercurio [Mer05] formulate the calibration problem for the general HJM framework in an abstract way and concentrate on swaptions and caps. Unfortunately, they neither discuss the practicability nor analyze the numerical tractability. Andersen and Andreasen [And02] identify several problems concerning the numerical tractability of the general calibration problem for the Gaussian HJM model. Especially the computation time of pricing vanilla caps and swaptions complicates the accurate calibration. They therefore suggest an approximation by asymptotic expansions and apply it to low-dimensional HJM models. They receive accurate results for small maturities, but for long maturities these worsen sometimes. A discussion of the used optimization algorithms or the applicability of known techniques is missing as well. A slightly different approach is presented by Andreasen [And05], who approximates the underlying stochastic differential equation by a time-homogeneous model. Thus he receives (quasi) closed-form pricing formulas. This type of asymptotic solution just takes simple volatility structures into account and does not deliver convincing results. Again Andreasen

[And00] suggests another approach to simplify the calibration especially to swaptions. He approximates the dynamic of the swap rate, which makes the bootstrapping of the volatility possible. The method is based on the pricing PDE and uses Finite Differences techniques to compute the prices. The algorithm is really fast, but it can only be applied to really easy volatility parametrizations in low-dimensional HJM models so far.

All the presented approaches incorporate an approximation of the pricing formula to simplify the minimization problem. In this paper we use the exact pricing formulas developed by [Hen03] and analyze several optimization techniques to receive accurate results in reasonable computation time. Thus, we do not simply adjust the problem, but apply different optimization techniques to solve it.

3 The Cheyette Interest Rate Models

3.1 Derivation of the Model

The Cheyette interest rate model is a specialization of the general HJM framework, so that we will present the general setup first. In the second step we will limit the class of models to a specific process structure of the yield curve dynamic. This technique will guide us to the representation of the class of Cheyette models.

3.1.1 The Heath-Jarrow-Morton framework

The Heath-Jarrow-Morton approach yields a general framework for evaluating interest-rate derivatives. It can be classified as a forward-rate model, in contrast to short rate models or market models. The uncertainty in the economy is characterized by the probability space $(\Omega, \mathcal{F}, Q)$, where Ω is the state space, $\mathcal{F}$ is the σ-algebra representing measurable events, and Q is a probability measure. The dynamic of the forward rate is given by an Itô process

$$\mathrm{d}f(t,T) = \mu(t,T)\mathrm{d}t + \sigma(t,T)\mathrm{d}W(t). \tag{1}$$

It is assumed that W(t) is an m-dimensional (Q-)Brownian motion and the drift $\mu = (\mu(t,T))_{t\in[t_0,T]}$ and the volatility of the forward rate $\sigma = (\sigma(t,T))_{t\in[t_0,T]}$ are m-dimensional progressively measurable stochastic processes satisfying certain conditions on the regularity:

$$\int_{t_0}^{T} |\mu(s,T)|\mathrm{d}s < \infty \quad Q-a.s.\ \forall t_0 \le T \le T^*, \tag{2}$$

$$\int_{t_0}^{T} \sigma_j^2(s,T)\mathrm{d}s < \infty \quad Q-a.s.\ \forall t_0 \le T \le T^*, j=1,...,m, \tag{3}$$

$$\int_{t_0}^{T^*} |f(t_0,s)|\mathrm{d}s < \infty \quad Q-a.s., \tag{4}$$

$$\int_{t_0}^{T^*} \left(\int_{t_0}^{u} |\mu(s,u)|\mathrm{d}s \right) du < \infty \quad Q-a.s.. \tag{5}$$

The notations use the fixed maturities $T \geq t$ as well as the maximum time horizon $T^* \geq T$. Then the forward rate is given by

$$f(t,T) \;=\; f(0,T) + \int_{t_0}^{t} \mu(s,T)\mathrm{d}s + \int_{t_0}^{t} \sigma(s,T)\mathrm{d}W(s). \tag{6}$$

The construction of the forward rate structure is based on the assumption of a complete market. The economic concept of a complete market can be translated to the existence of a unique martingale measure. This condition can, for instance, be fulfilled by assuming a restriction on the forward rate drift $\mu(t,T)$.

Condition 3.1 (Forward rate drift restriction)**.**
If and only if the drift of the forward rate has the structure

$$\mu(t,T) = \sigma(t,T) \left(\int_{t}^{T} \sigma(t,v)\,\mathrm{d}v - \lambda(t) \right) \qquad \forall T \in [0,T^*] \text{ and } t \in [0,T],$$

whereas $\lambda(t)$ notes the Market Price of Risk*, then an equivalent martingale measure exists .*

The implied forward rate under the drift restriction is given by

$$f(t,T) = f(0,T) + \int_{t_0}^{t} \sigma(s,T) \left(\int_{s}^{T} \sigma(s,v)\mathrm{d}v - \lambda(s) \right) ds + \int_{t_0}^{t} \sigma(s,T)\mathrm{d}W(s).$$

In the following, we will neglect the market price of risk and set $\lambda(t) \equiv 0$, so that the forward rate reduces to

$$f(t,T) = f(0,T) + \int_{t_0}^{t} \sigma(s,T) \left(\int_{s}^{T} \sigma(s,v)\mathrm{d}v \right) ds + \int_{t_0}^{t} \sigma(s,T)\mathrm{d}W(s). \quad (7)$$

Remark 3.2.
If the volatility $\sigma(t,T)$ is a deterministic function, then the forward rate and the short rate are normally distributed.

Remark 3.3.
The derivation of the forward rate dynamics is only based on the forward rate drift restriction, so that the HJM framework contains a really wide class of models. As a consequence, one can state that the term structure of volatility determines the forward rate at all times.

In order to close the derivation of the general HJM setup, we sum up the general HJM approach as a model having essential theoretical advantages and follow [KL07].

(i) The yield curve dynamics are completely determined once the structure of (forward rate) volatility is specified and the initial market yield curve data is included.

(ii) By specifying the volatility structure in this general class of yield curve models, one recovers short-rate models or the Libor Market Model.

(iii) To make the general HJM model consistent with market practice, which is dominated by Black's formula and assumption on the forward rate dynamics, we would assume the forward rate to be lognormally distributed. One straightforward approach is given by setting

$$\sigma(t,T) = const \cdot f(t,T)$$

.

3.1.2 The Cheyette Model

The class of Cheyette interest rate models forms a subset of the general class of HJM models. As already suggested in the literature, one can choose a specific volatility structure $\sigma(t,T)$ and achieves an exogenous model of the yield curve with Markovian dynamics. We will follow the ansatz of O. Cheyette

[Che94] and use a separable volatility term structure. The volatility function is assumed to be separable into time and maturity dependent factors given by the structure

$$\sigma(t,T) = \alpha(T)\frac{\beta(t)}{\alpha(t)}. \tag{8}$$

Following this idea, one arrives at a subclass of yield-curve models, where the risk-neutral HJM dynamics can be represented through a system of Markovian SDEs. The models implied by (8) do not cover all possible forward rate structures, e.g. a humped shape can not be generated. In order to increase the complexity of the class, we can extend the volatility term structure to a finite sum of separable functions and allow

$$\sigma(t,T) = \sum_{i=1}^{N} \sigma^{(i)}(t,T) = \sum_{i=1}^{N} \alpha_i(T)\frac{\beta_i(t)}{\alpha_i(t)}. \tag{9}$$

The choice of the volatility structure affects the characteristics of the covered models. Even this abstract definition implies the following changes:

(i) In the general HJM framework the state space of volatility functions is not finite. As a consequence of the reduction to the specific volatility term structure (9) the space becomes finite.

(ii) The dynamic of the forward rate will be determined by the short rate and the cumulative quadratic variation.

Remark 3.4.
As already mentioned, the class of Cheyette Models forms a subset of the class of HJM Models. The extent of the set of Cheyette models is determined by the set of supported volatility functions. Consequently, we can interpret the class of the Cheyette models as an approximation to the set of HJM Models. Formally, it is an approximation of a continuous function, the volatility and the number of summands N in equation (9) controls the accuracy. The theorem of Stone-Weierstrass states that the approximation is uniform. Empirical research [Che94] has shown that already two summands deliver sufficiently accurate results.

The dynamic of the forward rate can be reformulated as follows, if we assume the mentioned volatility structure:

$$f(t,T) = f(0,T) + \sum_{j=1}^{N} \frac{\alpha_j(T)}{\alpha_j(t)} \left[x_j(t) + \sum_{i=1}^{N} \frac{A_i(T) - A_i(t)}{\alpha_i(t)} V_{ij}(t) \right]. \tag{10}$$

The presentation uses the following notations for $i, j = 1, ..., N$:

$$A_k(t) = \int_0^t \alpha_k(s)\mathrm{d}s, \tag{11}$$

$$x_i(t) = \int_0^t \frac{\alpha_i(t)}{\alpha_i(s)}\beta_i(s)\,\mathrm{d}W(s) + \int_0^t \frac{\alpha_i(t)\beta_i(s)}{\alpha_i(s)}\left[\sum_{k=1}^{N}\frac{A_k(t)-A_k(s)}{\alpha_k(s)}\beta_k(s)\right]\mathrm{d}s, \tag{12}$$

$$V_{ij}(t) = V_{ji}(t) = \int_0^t \frac{\alpha_i(t)\alpha_j(t)}{\alpha_i(s)\alpha_j(s)}\beta_i(s)\beta_j(s)\mathrm{d}s. \tag{13}$$

The dynamic of the forward rate is determined by the state variables $x_i(t)$ and $V_{ij}(t)$ for $i, j = 1, ..., N$. The stochastic variable x_i describes the short rate and the non-stochastic variable V_{ij} states the cumulative quadratic variation. Summarizing, the forward rate is determined by $\frac{N}{2}(N+3)$ state variables. The dynamics of the short rate and the quadratic variation are given by Markov processes as

$$\mathrm{d}x_i(t) = \left(x_i(t)\partial_t(\log\alpha_i(t)) + \sum_{k=1}^{N} V_{ik}(t)\right)\mathrm{d}t + \beta_i(t)\mathrm{d}W(t) \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}V_{ij}(t) = \beta_i(t)\beta_j(t) + V_{ij}(t)\partial_t(\log(\alpha_i(t)\alpha_j(t))). \tag{15}$$

The Cheyette Model and all features were achieved just by assuming a special structure for the volatility and on the basis of the assumptions of the general HJM framework. The first variable can be interpreted as a yield curve factor and the second as a convexity adjustment term ensuring that the model is arbitrage-free. The representation of the forward rate (10) delivers several advantages concerning the numerical tractability:

(i) The differential equations describing the dynamic of the state variables (14) and (15) can be solved independently. Due to this improvement, the simulation of the forward rate becomes more efficient.

(ii) The SDE (14) is a Markov Process and by using the Feynman-Kac Theorem, it can be transformed to a PDE. Several well known numerical methods like Finite Elements or Finite Differences can thus be applied to solve it.

At this stage, the Cheyette Model is defined as a one factor model, but it can easily be generalized to a multi-factor model. The additional factors are given by several independent Brownian motions and the forward rate is given by

$$f(t,T) = f(0,T) + \sum_{i=1}^{M} \tilde{f}^i(t,T), \tag{16}$$

where $\tilde{f}^i(t,T)$ denotes a one factor forward rate defined by (10).

3.2 Volatility Parametrization

In the Cheyette Model, the volatility function is assumed to be the finite sum of separable functions given by (9),

$$\sigma(t,T) = \sum_{i=1}^{N} \sigma^{(i)}(t,T) = \sum_{i=1}^{N} \frac{\alpha_i(T)}{\alpha_i(t)} \beta_i(t).$$

Empirical results [Che94] propose a volatility function with two or three summands to achieve a reasonable structure including a permanent humped shape. One summand should form a constant shift of the structure and consequently we choose

$$\sigma^{(1)}(t,T) = c \equiv const. \tag{17}$$

The value of the constant does not affect the shape of the volatility function, it just adds a constant shift. This representation can be achieved by choosing $\alpha_i(t) = 1$ and $\beta_i(t) = c$, which are obviously separable functions. The shape of the volatility structure is determined by the other summands taken into account. In the academic literature, e.g [Che94], an exponential structure of $\alpha_i(t)$ is promoted, implying

$$\sigma^{(i)}(t,T) = \beta_i(t) \exp\left(- \int_t^T \kappa_i(u) du \right). \tag{18}$$

The implied model is called *exponential Cheyette Model*. This suggested structure again includes two degrees of freedom, namely the deterministic functions $\beta(t)$ and $\kappa(u)$. The most intuitive and easiest choice is assuming $\kappa(u)$ to be constant. Thus the parametrization reduces to

$$\sigma^{(i)}(t,T) = \beta_i(t) \exp\left(-(T-t)\kappa_i\right). \tag{19}$$

Assuming this structure, only $\beta(t)$ remains as a time-dependant parameter. As it is a function of time and not of maturity, $\beta(t)$ cannot control the volatility with respect to the remaining lifetime $(T-t)$. This factor can be addressed by $\alpha(t)$. Assuming κ to be constant implies no flexibility of the volatility structure with respect to the remaining lifetime. This might be a disadvantage in fluctuating markets.

This motivates us to allow $\kappa(u)$ to be piecewise constant. Therefore, we install some nodes dividing the domain of $\kappa(u)$. Depending on the simulated time horizon, we implement, for example, nodes for $1, 3, 5, 7,$ and 10 years. $\kappa(u)$ is assumed to be constant within each time interval. On the one side this extension improves the lack of flexibility with respect to the remaining lifetime, on the other hand it increases the number of degrees of freedom for the volatility parametrization. Instead of one parameter (κ constant), we have to determine six values (κ piecewise constant with five nodes). The values of the parameters are determined by the calibration of the model in section 4 . The calibration is equivalent to a (global) minimization and the boost of free parameters increases the dimension of the optimization problem as well.

In addition to the structure of $\kappa(u)$ one can influence the range and flexibility of the volatility structure by determining $\beta(t)$. The simplest choice is assuming $\beta(t)$ to be constant. Unfortunately this leads to a time-independent scaling factor and from our experience with various tests the range of the resulting volatility function is not wide enough for most of the applications. The results are improved by assuming $\beta(t)$ to be a linear or a quadratic function of time, i.e.

$$\beta(t) = at + b \quad \text{or} \tag{20}$$

$$\beta(t) = at^2 + bt + c. \tag{21}$$

Figure 1 and Figure 2 show possible volatility surfaces in the domain $[0, 10] \times [0, 10]$ representing the current time t and the remaining lifetime T. Figure 1 compares volatility surfaces for a linear function$\beta(t) = at + b$ combined with, on the one hand, a constant $\kappa(t) = \kappa$ (blue surface) and, on the other hand, a piecewise constant function $\kappa(t)$ (red surface). The coefficients of the linear function (20) are chosen arbitrarily, but equal in both structures. The plot demonstrates that the second volatility structure ($\kappa(t)$ piecewise constant) supports more flexibility and movements of the parametrization. Especially the shapes in direction of the remaining lifetime T are more flexible. The first structure ($\kappa(t) = const.$) just supports constant changes with respect to the remaining lifetime and thus it appears

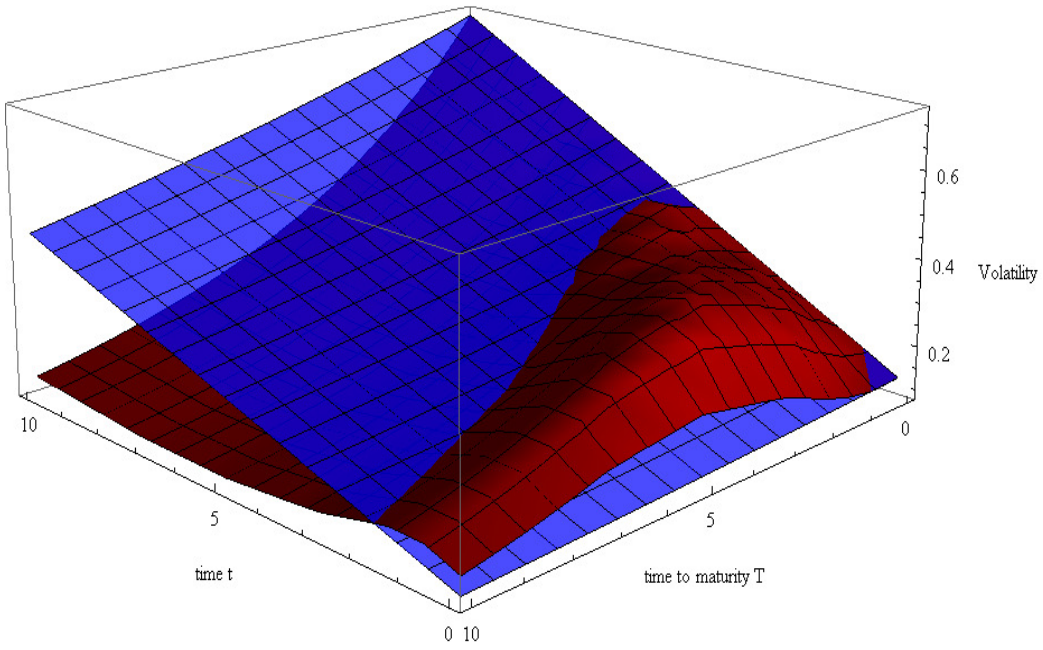

Figure 1: Representation of possible volatility surfaces of the exponential Cheyette model with $\beta(t)$ as a linear function (20) and a constant κ (blue) and a piecewise constant κ (red). The constant shift in both structures is given by $c = 0.03$. The coefficients of $\beta(t)$ take the values $a = 0.06$, $b = 0.1$. The coefficients of the piecewise constant κ are given by $\kappa_1 = 0.06$, $\kappa_2 = -0.3$, $\kappa_3 = -0.1$, $\kappa_4 = 0.07$, $\kappa_5 = 0.2$ and the constant κ is chosen as $\kappa = 0.05$.

rather rigid. One crucial point within the volatility function is the behavior for short lifetimes. The market often implies large movements in this section in comparison to volatilities for longer lifetimes. As demonstrated, the second model is superior concerning this aspect.

Figure 2 compares volatility structures taking two and three summands of separable functions into account. The parametrization with two summands consists of a linear function $\beta(t)$ and a piecewise constant $\kappa(t)$ in the exponential part. Additionally, the more complex model takes the quotient of remaining lifetime and current time linearly into account:

$$\sigma_T^{(3)}(t,T) = c\frac{T}{t}.$$

This implies more flexibility especially for small t and bears some advantages in extreme markets. Figure 2 shows possible structures and to emphasize the differences we fix the time to maturity $T = 5.0$. Thus the plot depicts a slice

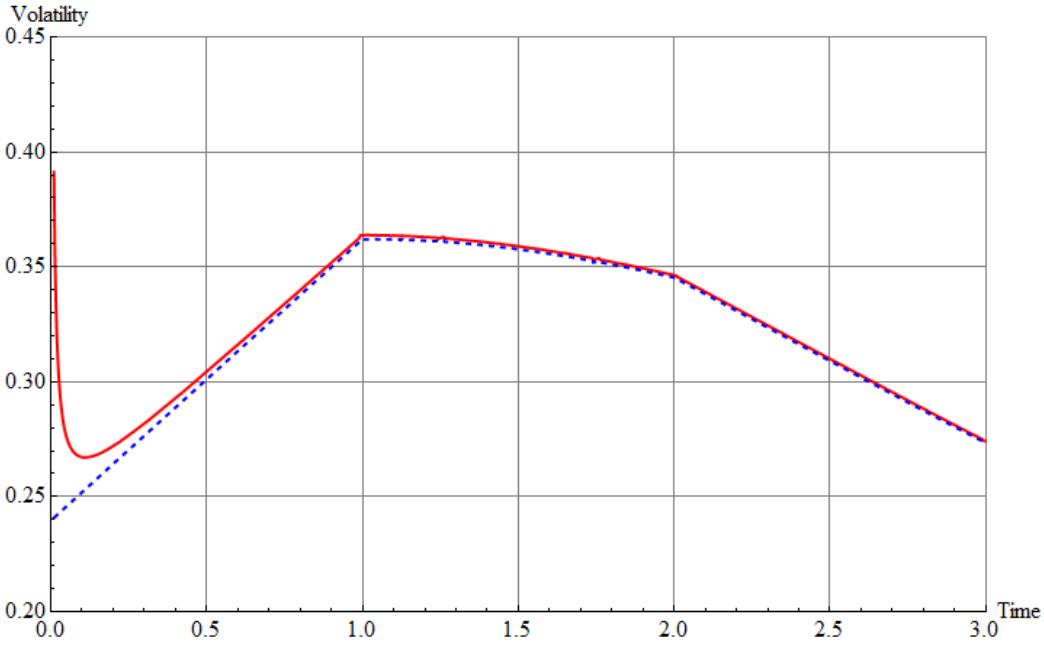

Figure 2: Representation of possible volatility structures consisting of two and three summands: constant shift, exponential form with linear β and piecewise constant κ and quotient for remaining lifetime. The plot shows a slice plane of the surface at $T = 5.0$. The coefficients are chosen as in Figure 1 and the additional scaling factor of the quotient for the remaining lifetime equals $c = 0.0003$.

plane of the volatility surface.

Furthermore, we tested several different choices of $\beta_i(t)$ and one interesting alternative takes time logarithmically into account, i.e.

$$\beta(t) = a\ln(t)^2 + b\ln(t) + c. \tag{22}$$

If derivatives with extremely varying lifetime are used simultaneously, this structure especially bears several advantages and leads to significantly better results. A possible volatility surface based on this structure is exemplarily shown in Figure 3 .

3.3 Pricing Formulas

The interest rate models are based on several assumptions, hence each model implies different formulas for pricing plain derivatives and exotic products.

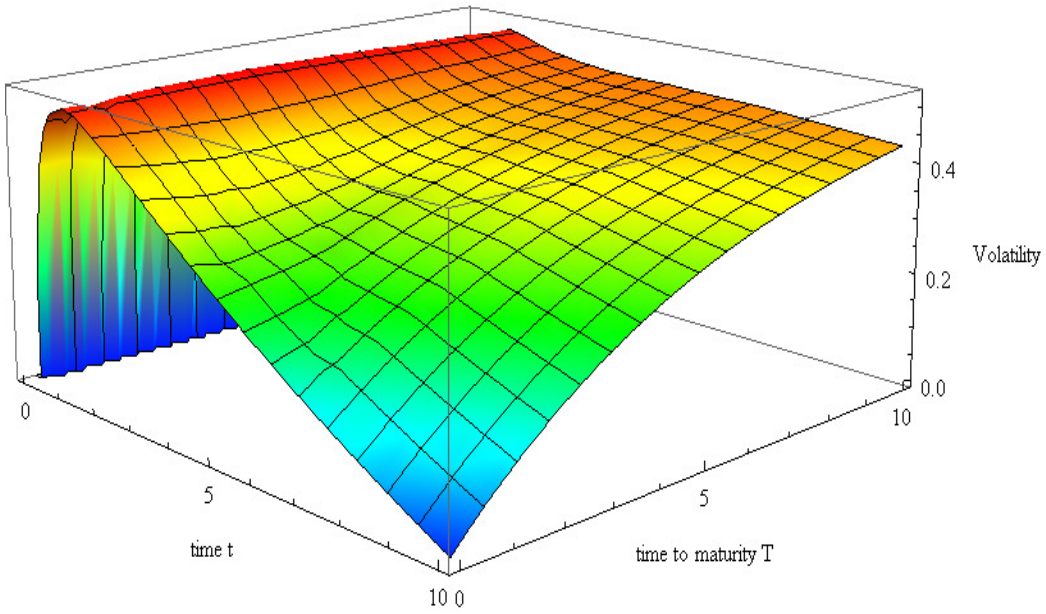

Figure 3: Possible volatility surface with two summands and logarithmical time scaling. The model structure and the coefficients are the same as in Figure 1 .

The models in the class of HJM and especially the Cheyette models are based on the same assumptions and differ only in the volatility function. Henrard [Hen03] gives semi-explicit formulas for pricing options on bonds and swaptions in the general one factor HJM framework. These formulas are based on a general structure of the volatility and if we choose the volatility according to the assumptions in the Cheyette Model (9), we derive pricing formulas for these type of interest rate models. The calibration of the models incorporates the pricing formulas for at-the-money caps, floors and European swaptions.

Theorem 3.5 (Swaption pricing formula)**.**
The present value (PV) for (receiver) at-the-money swaptions in the Cheyette model is given by the semi-implicit formula

$$PV = \sum_{i=1}^{n} c_i P(0,t_i)\mathcal{N}(\lambda+\alpha_i) - P(0,t_0)\mathcal{N}(\lambda+\alpha_0). \tag{23}$$

λ is the unique solution to

$$\sum_{i=1}^{n} c_i P(0,t_i) e^{-\frac{1}{2}\alpha_i^2 - \alpha_i \lambda} = P(0,t_0) e^{-\frac{1}{2}\alpha_0^2 - \alpha_0 \lambda} \tag{24}$$

and α_i *is defined by*

$$\begin{aligned} \alpha_i^2 &= \int_0^\theta \left(\int_s^{t_i} \sigma(s,x)dx - \int_s^\theta \sigma(s,x)dx \right)^2 ds \\ &= \int_0^\theta \left(\int_\theta^{t_i} \sigma(s,x)dx \right)^2 ds \end{aligned}$$

whereby $\sigma(t,T)$ *denotes the volatility at time* t *with maturity* T*. The realisation of volatility functions depends on the choice of the degrees of freedom in the parametrization summarized in the vector* $\phi \in \mathbb{R}^d$*. As the values of* α_i *and* κ *are determined by the volatility, these values and the resulting present value depend on* $\phi \in \mathbb{R}^d$ *as well.*

Furthermore we use the notations:

- $\mathcal{N}(.)$ *standard normal cumulative distribution function with mean* 0 *and standard deviation* 1
- $\theta < t_0$ *maturity of the option, the difference is about* 2 *business days and thus we approximate* $\theta \approx t_0$
- *payment dates (in years) in the swap* $t_0 \approx \theta, t_1, ..., t_n$
- *coupon payment* c_i *at date* t_i*:* $c_i = strike \quad \forall i < n; \quad c_n = 1 + strike$
- $P(0,t_i)$ *discount factor at time* 0 *with maturity* t_i

The pricing formula for caps and floors can be created by dividing the payoff structure into smaller units. Following Hull [Hul05], one can divide a cap/floor into a portfolio of European put/call options on zero coupon bonds with adjusted payoff and strike. Applying this decomposition one can reduce the pricing of caps/floors to the pricing of several options on zero bonds. Similar to the pricing of swaptions, Henrard [Hen03] introduces explicit formulas for options on bonds.

Theorem 3.6 (Pricing Formula for Options on Bonds)**.**
Let $\theta \leq t_0 \leq t_1 \leq ... \leq t_n$. Consider a bond which pays c_i at times t_i $(1 \leq i \leq n)$. At time 0, the price of a European call on the bond with expiry θ and strike price K to be paid in t_0 is

$$\sum_{i=1}^{n} c_i P(0,t_i) N(\lambda + \alpha_i) - KP(0,t_0) N(\lambda + \alpha_0)$$

where λ is the (unique) solution of

$$\sum_{i=1}^{n} c_i P(0,t_i) \exp(-\frac{1}{2}\alpha_i^2 - \alpha_i \lambda) = KP(0,t_0)$$

and the α_i's are the positive numbers such that

$$\alpha_i^2 = \int_0^{\theta} \left(\int_{\theta}^{t_i} \sigma_T(s,x) dx \right)^2 ds.$$

The price of a European put is given by

$$KP(0,t_0) N(-\lambda - \alpha_0) - \sum_{i=1}^{n} c_i P(0,t_i) N(-\lambda - \alpha_i).$$

In case of a zero coupon bond the presented formula reduces in case of a zero coupon bond to a single payment and thus λ is given explicitly by

$$\lambda = \frac{-1}{\alpha_1} \ln \frac{KP(0,t_0)}{c_1 P(0,t_1)} - \frac{1}{2}\alpha_1.$$

Based on these pricing formulas we can formulate the minimization problem for the calibration. The existence and the uniqueness of λ for the general case is shown in [Hen03].

4 The Calibration Problem

4.1 Formulation

Dealing with term structure models, the possibility to represent the current state of the financial market is one of the most important characteristics. This feature can be obtained by creating flexible models that could generate any shape of the term structure. The aimed flexibility can be achieved by

term structure models taking several (stochastic) factors into account. Each factor represents a predefined shape, e.g. parallel shift or twist. As a consequence the models become more complex and the mathematical tractability decreases. In sum, the model should rebuild a reasonable fraction of the financial market at best and at the same time stay as simple as possible. The quality of the approximation of the market can be measured by comparing characteristic numbers of standardized financial instruments quoted in this market fraction, e.g. prices or volatility. A common method to measure the accuracy of the approximation is based on the differences in pricing between the observed market prices and the model implied prices. Changing the point of view slightly, we can look for a term structure model minimizing these differences for a specific market section, i.e. we calibrate a model to a specific market section. The formulation of the calibration problem is based on the degrees of freedom within the specific structure. The number of free model parameters depends on the number of included factors, so the complexity of the model appears proportional to this number.

Cheyette models have a special volatility term structure that is parameterized by a sum of separable functions. As a consequence the major processes have the Markov property that forms an advantage for the subsequent numerical simulation. The set of free parameters in the model is limited to the coefficients representing the parametrization of the volatility.

The calibration of the Cheyette model is performed on the basis of several at-the-money European swaptions (receiver) with different maturities up to ten years and caps/floors (at the money). The prices of the derivatives as quoted in the market (based on the Black-Scholes model) are compared to the prices computed in the model depending on coefficients $\phi \in \mathbb{R}^d$. The market values of the derivatives are assumed to be computed in the Black-Scholes model and the corresponding pricing formulas for swaptions and caps/floors are one-to-one. They can thus be inverted locally with respect to the volatility. Instead of comparing the price differences as presented, we can therefore compare the differences in the implied Black-Scholes volatilities. The comparison of the implied volatility should be preferred, because the level of values is standardized as opposed to the prices. A comparison is thus more meaningful. Following this ansatz, the calibration problem is given by

Problem 4.1.
Assume $\Phi \subset \mathbb{R}^d$ to be the set of free parameters in the model and further $PV : \Phi \subset \mathbb{R}^d \to \mathbb{R}$ to be a pricing formula (e.g. for swaptions) depending on the degrees of freedom. Furthermore, let $\tilde{\sigma}_i$ denotes the implied Black-

Scholes volatility as observed in the market and $\sigma_i(PV_i(\phi))$ *the implied Black-Scholes volatility representing the price* $PV_i(\phi)$ *in the Cheyette model with the parameter set* $\phi \in \Phi$*. In the calibration process we are looking for the global minimum of the function* $E : \mathbb{R}^d \to \mathbb{R}$*.*

$$\inf_{\phi \in \Phi} E(\phi), \qquad E(\phi) = \sum_{i=0}^{I} \omega_i |\sigma(PV_i(\phi)) - \tilde{\sigma}_i|^2. \tag{25}$$

The integer I denotes the number of all derivatives taken into account for the calibration and $\omega_i \in \mathbb{R}$ denotes weights that control the influence of any instrument; for example swaptions and caps might be weighted differently.

4.2 Constraints

The optimization does not include any explicit constraints on the single parameters. Instead, the reasonable constraint

$$\sigma(t,T) \geq 0 \qquad \forall t > 0, T \geq t \tag{26}$$

should be fulfilled by the interaction of all coefficients in the parametrization of the volatility.

4.3 Characterization of the optimization space

The optimization space is determined by the realizations of the function $E : \Phi \subset \mathbb{R}^d \to \mathbb{R}$. The search for the global minimum strongly depends on the characteristics of the function E. Obviously E is non-linear and the formula for the (Black-Scholes) implied volatility based on the present values in the Cheyette Model is not given explicitly. One even has to perform a one-dimensional Newton optimization to compute these values. Furthermore, the corresponding minimization problem appears mostly high dimensional, because it is linked to the choice of volatility parametrizations. Often it contains more than eight free coefficients. In addition to the non-linearity, the function is generally neither convex nor concave, which complicates the optimization problem dramatically and many methods are no longer applicable. Only in very simple cases, like a constant volatility, the function is monotone.

The intractability of the function is even intensified by the fact that the function is not monotone for all coefficients. Only if we fix all coefficients but one, the function appears to be monotone in this variable. The combination of variables destroys this feature. To sum up, the function does not hold

many helpful features to simplify the optimization problem.

Due to the complex structure of the function, especially the non-explicit formula for the implied volatility, the computation of a single function value takes about 0.1 sec[1] on average and is rather time consuming. Several optimization methods are based on (partial) derivatives, which are not applicable in this case. The use of difference quotients does not produce stable results either. The difference quotient often vanishes, if one of the coefficients is far from the minimum, because small changes might not influence the value of the function. Several plateaus with the same function value appear and hence the difference quotient must vanish.

In the following, we will show some examples of possible spaces for a given state of the market. As the space represents the differences in the implied volatility between the market values and the model implied values, the space depends on the used state of the market and changes dynamically. As the space in the example is five-dimensional, we have to use an appropriate projection for the graphical presentation. The presentation is based on the market data observed at the end of June 2009 and it just takes (at-the-money) swaptions into account. We take 10 swaptions with option lifetime of 3 months and 1 year each combined with swap lifetimes of 2, 3, 4, 5 and 6 years. The Cheyette Model used is determined by the volatility parametrization

$$\sigma(t,T) = c + (at+b)\exp\left(-\int_t^T \kappa(u)du\right). \tag{27}$$

The deterministic function $\kappa(u)$ is assumed to be piecewise constant and the domain is divided into two disjoint intervals. Thus there are five free parameters. This structure is rather crude, but it shows most of the characteristics of the minimization surface. As the coefficient $c \in \mathbb{R}$ just adds a constant shift, the volatility structure is determined by a, b, κ_1 and κ_2.

Figure 4 and Figure 5 show the differences in implied (Black-Scholes) volatility for swaptions between the observed market values and the model implied values for a given parameter set $\phi \in \Phi$. The sample takes 10 swaptions into account and the error is given as the sum of the squared differences. In order to demonstrate the structure of the error surface, we vary two coefficients in each plot and present the resulting error surface. The remaining

[1]We used a Windows based PC with Intel Core 2 Duo CPU @ 1.66 GHz and 3.25 GB RAM.

coefficients are held constant in each graph.

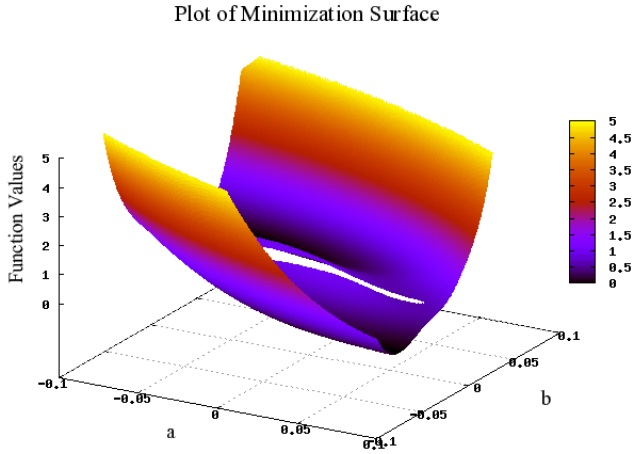

Figure 4: Minimization surface with respect to the variables a and b in the calibration process of the model implied by equation (27)

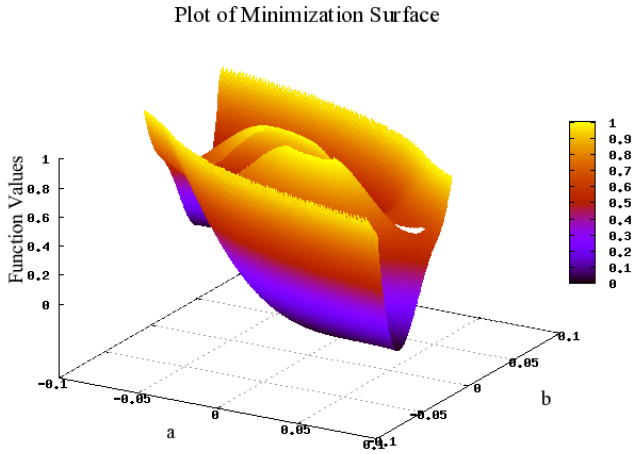

Figure 5: Zoom into the minimization surface given in Figure 4

Figure 4 and Figure 5 display the movement of the error surface with respect to the coefficients a and b. The interval $[-0.1, 0.1]$ for possible values is suggested by empirical tests. The surface contains several holes, due to the range of possible values of the z-axis. Each hole in the surface implies

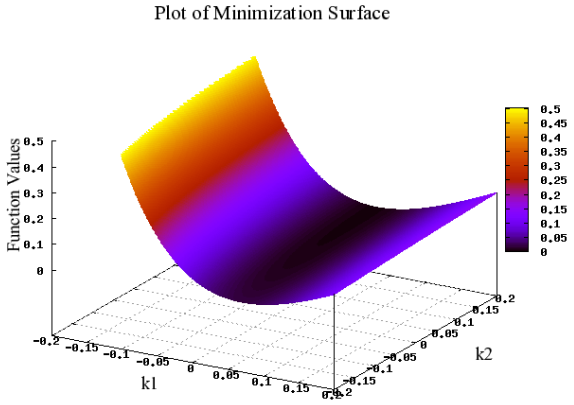

Figure 6: Minimization surface with respect to the variables κ_1 and κ_2 in the calibration process of the model implied by equation (27)

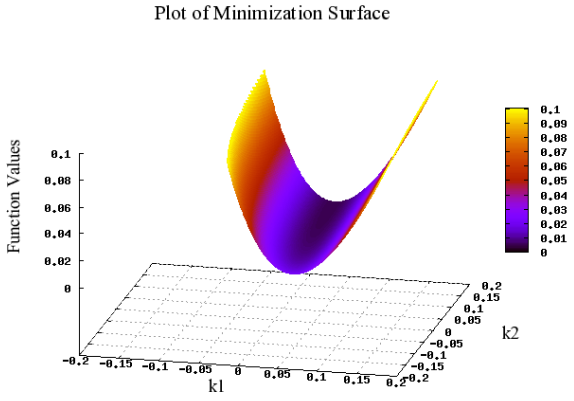

Figure 7: Zoom into the minimization surface given in Figure 6

an error value exceeding the maximal range. Consequently, there are several jumps and the surface is not continuous. This is caused by the numerical computation of the implied (Black-Scholes) volatility, because a calculated value exceeding the limit of 100% volatility is forced to take a high technical value. Hence, if one value implied by the model exceeds a predefined level, a hole will be created in the used plot.

In Figure 4 there are two separate regions possibly containing the (global) minimum. At the border and in the middle of the analyzed interval for the coefficients, the error values increase rapidly. As the range of the z-axis is

rather big, we zoomed in and displayed the same surface in Figure 5 for a smaller range of values. This figure demonstrates that there are valleys in the surface and that the error reacts sensitively to small changes in these coefficients. Figure 6 and Figure 7 display the movement of the error surface with respect to the coefficients κ_1 and κ_2 of the piecewise constant function $\kappa(u)$. The remaining coefficients are fixed and chosen to minimize the error for the given data set. The surface appears rather regular and contains no hills. The structure of the surface suggests that the choice of $\kappa(u)$ does not change the values dramatically. But the pictured surfaces only include two degrees of freedom for illustration. In general, $\kappa(u)$ takes at least five coefficients into account and thus several valleys, including (local) minima, emerge.

Each of the presented surfaces only depends on two parameters. The whole surface is given as a combination of these structures. Thus the resulting surface becomes more turbulent and irregular. Consequently, there are several local minima, jumps and hills, that have to be treated in the optimization process. Several methods that can handle these functions exist, but none of them guarantees the convergence to a global optimum.

4.4 Quality Check

The evaluation of the calibration results leaves much freedom to the user. In order to apply a standardized technique, we come up with several conditions to the prices and the implied Black-Scholes volatility to evaluate the calibration. We focus on the price and volatility residuals between the Cheyette and the Black-Scholes model. The implemented check of the calibration consists of twelve conditions subdivided into four core and eight secondary ones. The design and the definition of the quality check is comparable to the criteria used in the pricing software 'FINCAD Analytics Suite 2009'. The core conditions are given by:

- Price Bias $\leq 30\%$:
  At most 30% of the relative price differences exceed the limit of 30%
- Volatility Bias $\leq 30\%$:
  At most 30% of the relative volatility differences exceed the limit of 30%
- Mean price residual $< \sigma$:
  The mean of all price residuals does not exceed the standard deviation σ with mean 0

- Mean volatility residual $< \sigma$:
  The mean of all volatility residuals does not exceed the standard deviation σ with mean 0

Furthermore we identified several secondary conditions given by:

- Convergence of the minimization algorithm
- Minimum does not lie on the boundary
- All volatility residuals are bounded by 3σ
- All price residuals are bounded by 3σ
- 90% of the volatility residuals are less than 2σ
- 90% of the price residuals are less than 2σ
- Maximal volatility residual does not exceed 5%
- Mean of squared volatility residuals is limited by 1

The quality check delivers three possible outcomes: good, passed and failed. If all conditions - core and secondary - are fulfilled, the calibration is assumed to be 'good'. If at most one secondary condition does not hold, the calibration is interpreted to be still valid and thus the algorithm will deliver 'passed'. The calibration algorithm fails, if one or more of the core conditions or more than one of the additional conditions are violated.

5 Optimization Methods

5.1 Overview of methods

Developing the calibration method, we have implemented several minimization algorithms. Mainly, we can classify the tested algorithms into three different classes:

1. non-linear optimization methods with derivatives, e.g. Newton algorithm with and without step size adjustment,
2. non-linear optimization methods without derivatives, e.g. Downhill Simplex and Powell algorithm and
3. stochastic optimization methods, e.g. Simulated Annealing and Genetic Optimization.

5.2 Assessment of methods

In the following we present the analysis of the optimization methods consisting of method descriptions followed by the results of the application to the calibration problem.

5.2.1 Newton Algorithm

5.2.1.1 Description

The Newton algorithm is a local optimization method developed by Isaac Newton in 1669. This method makes use of the first and second derivatives with respect to all directions. The resulting Jacobi- and Hessian matrix are assumed to have full rank and furthermore the Hessian matrix has to be invertible. Based on these matrices, the iterations steps are computed. The algorithm will terminate in a position where the first derivatives vanish and thus the algorithm in general detects only local optima.

5.2.1.2 Results

The Newton algorithm as presented in [Sto03] is based on the Jacobi- and Hessian matrix including the first and second partial derivatives of the function to be minimized. The derivatives cannot be computed explicitly, so that one has to use difference quotients. The parametrization of the volatility implies that the partial derivatives vanish sometimes. The derivatives in direction of the variables κ (in case of the exponential model) vanish especially often, because changes in one variable do not have a huge effect on the price. Due to the vanishing partial derivatives, the Jacobi- and Hessian matrix have lower ranks and the Hessian matrix is no longer invertible. Consequently, the Newton algorithm does not converge. Convergence cannot be achieved by using a standard step size adjustment. Even if we vary the initial value, we cannot generate a stable algorithm.

5.2.2 Powell Algorithm

5.2.2.1 Description

The Powell algorithm is a powerful tool for minimizing non-linear functions and is, among others, presented in [Bre73]. The advantage of the Powell algorithm is based on the fact that no derivatives are necessary. The multidimensional (n dimensions) minimization problem can be divided into several $(n+1)$ one-dimensional problems. In order to reach an optimum in the

superior algorithm, we need to find the global optimum in each of the one-dimensional optimization problems. First, the one-dimensional minimization is performed separately along a line in direction of each volatility coefficient. In this case, the one-dimensional problem is easy, because the function appears monotone. Based on these results, new directions for the optimization are created by combining different one-dimensional optima. The new directions effect more than one coefficient of the volatility function. As a result, the monotonicity is lost and the one-dimensional problems becomes more complicated. The corresponding minimization space does not stay monotone any longer, but several waves will appear complicating the minimization. Due to this fact, we have to use a brute force algorithm testing more or less all possible values in a predefined interval. This method costs a lot of time, but we need to find a global minimum of the one-dimensional problem in order to reach the global minimum with the superior algorithm. We reinitialize the basis of directions to search after processing n iterations with the purpose to improve the Powell algorithm and decrease the linear dependency.

5.2.2.2 Results

The performance of the Powell algorithm depends strongly on the choice of the initial values. We tested the algorithm for the calibration to the whole swaption matrix and for the calibration to a fraction of the matrix. The calibration to the whole swaption matrix reaches the global optimum, if we start close to it. We have tested the algorithm with a known global optimum and the algorithm reached it quickly. Unfortunately, the algorithm is sensitive to the initial values, so that the algorithm does not find the global optimum if we start somewhere in the solution space. If we reduce the dimension of the problem by decreasing the number of coefficients, the algorithm seems to improve and produces better results. But this behavior cannot be observed in each case. In contrast to the calibration taking the whole swaption matrix into account, the reduction to a fraction of the matrix increases the accuracy of the solution. If we calibrate the model to a set of swaptions with the same fixed option lifetime and variable swap lifetime up to 10 years, we receive sufficiently accurate results. This means, the difference in the implied Black-Scholes volatility is less than 0.5% on average. This kind of calibration is more robust concerning the initial values, but if the algorithm starts far away of the global optimum, the convergence is not guaranteed.

The most important difficulty of the minimization is the different sensitiv-

ity of the coefficients to the minimization function. While, for example, the constant and the coefficients of the linear function (in the case of the example presented in (27)) have a strong effect on the minimization function, the effect of the coefficients of the piecewise constant function $\kappa(t)$ is relatively small. Thus one has to use different discretizations or step sizes for the coefficients to receive sufficiently accurate results and minimize iterations. The idea of the Powell algorithm includes the minimization along one-dimensional lines and the directions are composed by several coefficients. The discretization and the step size must thus be adapted to all coefficients which implies a large number of iterations and consequently large computation times. This is even intensified by the characteristics of the one-dimensional minimization problem possessing several waves. As the global optimum (along the line) is required only naive algorithms are available taking lots of time. In combination one single step in the Powell algorithm is time consuming and thus this algorithm is not efficient for the calibration of the Cheyette Model. The setup of the model carries undesired features destroying the advantages of the Powell algorithm for high dimensions.

5.2.3 Downhill Simplex Algorithm

5.2.3.1 Description

The Downhill Simplex method is an optimization method requiring only function valuations and no derivatives. It is not very efficient in terms of the number of function valuations, but it is robust and the results are reliable. The method is due to Nelder and Mead and was first published in 1965 [Mea65]. The simplex method takes an n-dimensional simplex consisting of $(n+1)$ points as initial value. Based on function valuations, the algorithm takes a series of steps. Most of the steps move the point of the simplex with the highest value to the opposite face of the simplex to a lower point. These steps are called reflections and are constructed to conserve the volume of the simplex. If it is possible to perform the reflections, the algorithm expands the simplex automatically to take larger steps. In case of a 'valley' the method contracts itself in the transverse direction and tries to escape the valley. The method continues as long as one of the terminal conditions is fulfilled. There are many possible conditions and we selected two: First, the algorithm stops if a cycle is identified and second, if the function value is sufficiently close to zero. The second termination criteria is only feasible, because we know that all values are positive and zero would imply a perfect fit.

It might happen that one of the above criteria is violated by a single

abnormality in the function. It is therefore a good idea to restart the minimization routine at a point where it claims to have found a minimum. The downhill simplex algorithm includes four parameters that control the movement of the simplex. The choice of these values influences the speed of the method and might change the computed optimum.

- α controls the reflection.
- β controls the contraction.
- γ controls the expansion.
- σ controls the compression.

Numerous publications mention only three parameters for the Downhill-Simplex algorithm and in this case they do not distinguish between parameter β and σ.

5.2.3.2 Results

We applied the Downhill Simplex algorithm to the calibration problem taking ten (at-the-money) swaptions into account. Furthermore, we tested three different parameter sets and analyzed the influence on the solutions. The parameter sets were chosen on the basis of a standardized calibration test with market data of 2006. Based on this data the three parameter sets deliver the best (set 1), the worst (set 3) and average (set 2) results as shown in Table 1 .

	α	β	γ	σ
Set 1	1.0	0.6	1.9	0.5
Set 2	1.2	0.7	2.2	0.3
Set 3	0.8	0.6	1.8	0.5

Table 1: Sets of analyzed parameters in the Downhill Simplex algorithm

In addition to changes in the parameters of the algorithm, we applied the calibration to different market data. We have used data from the end of Jan. 08, July 08, Jan. 09, July 09 and Jan. 10. Figure 8 shows the results for the time series and the different parameter sets. The plotted values are the minimal function values according to equation (25).

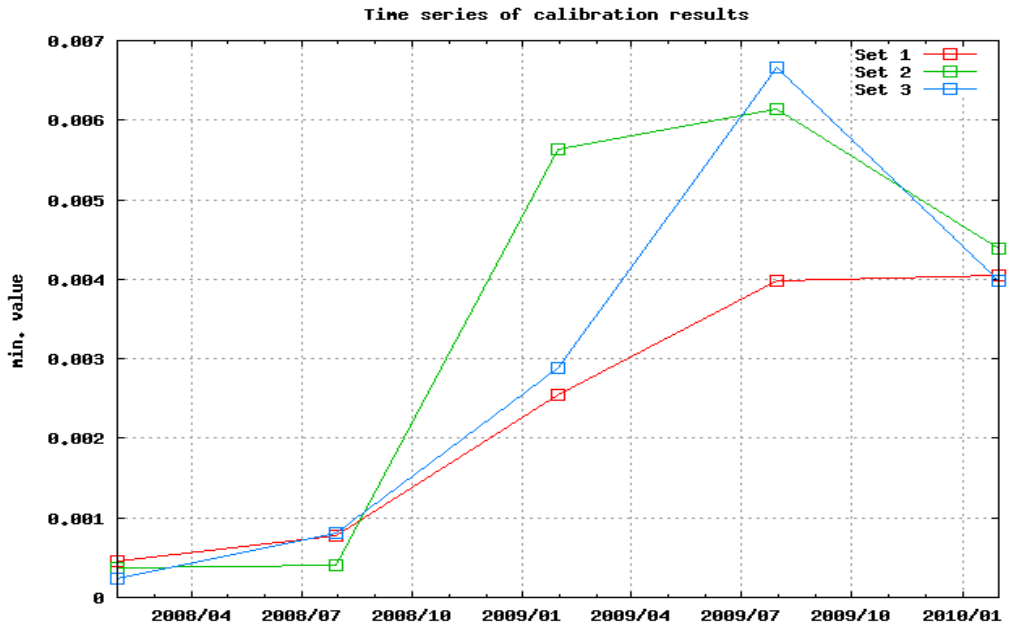

Figure 8: Time series of minimal values of the calibration to swaptions based on the Downhill Simplex algorithm and multiple parameter sets.

The computed minima of the time series vary between $2.4 \cdot 10^{-4}$ in Jan. 08 and $6.6 \cdot 10^{-3}$ in July 09 and the corresponding quality of the calibration results, according to section 4.4 , hold the best standards in all cases. The time series demonstrates that the calibration results are good in 2008. After that, the minimal function values worsen and imply that the proposed model does not fit the market data as well as in 2008. During that time, the level of swaption volatilities increases dramatically due to the financial crisis. The uncertainty in the market changed the structure of the volatility matrix. The calibration algorithm and the assumed model can build up these movements, but nevertheless the quality of the market reproduction decreases. But even the calibration results in 2009 fulfill the highest standards of the quality check.

Next to this, the changes of the market data during the crisis influence the behavior of the calibration with respect to the used parameter set. While the performance for the three sets only differ slightly in 2008, the range enlarges in 2009. Based on the outcomes, one can state that parameter set 1 is superior to the other ones. In 2008, set 1 does not deliver the best results, but the values are quite similar. Starting in 2009, when the results worsen, parameter set 1 is obviously dominant. Consequently, one should prefer this

setting, although it does not always produce the best results.

The presented effects are all based on the same initial values for the minimization algorithm. A significant criterion for reasonable optimization is the stability with respect to changes in the starting points. We tested 136 different initial values for the calibration and monitored the changes in the minimal values and in the coefficients of the minimum. The differences between the coefficients of two minima can be measured by the vector norm

$$|x^{(1)} - x^{(2)}| = \sum_{i=1}^{d} |x_i^{(1)} - x^{(2)_i}|^2 \tag{28}$$

and the differences between two minimal values by $|y^{(1)} - y^{(2)}|^2$. Figure 9 and Figure 10 show the histograms of differences in the minimal value by changing the initial points and the corresponding changes in the position given by the coefficients.

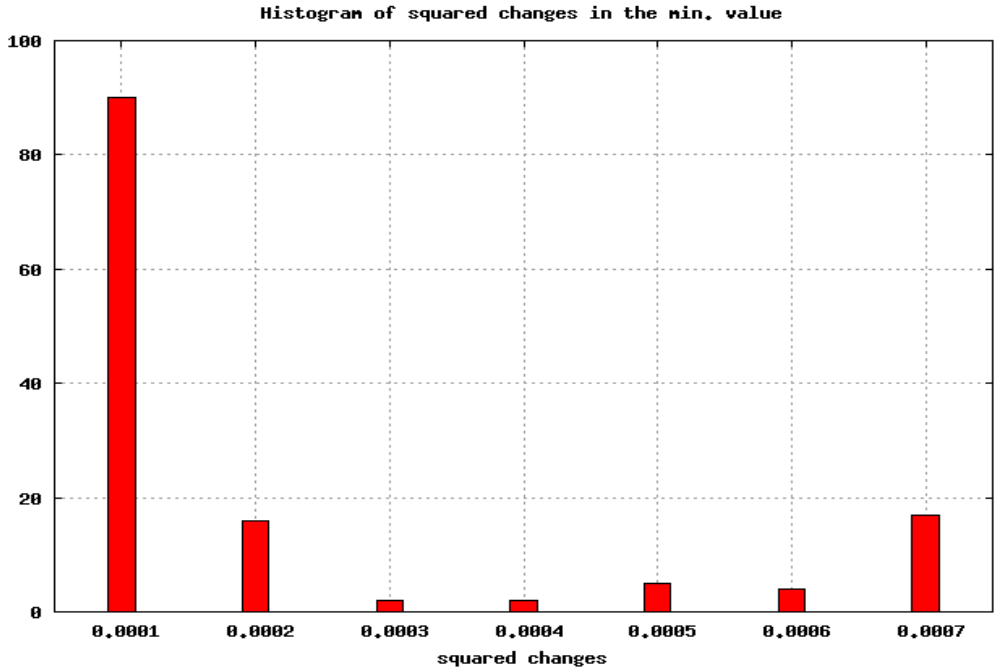

Figure 9: Histogram of changes in the minimal values for different initial values in the Downhill Simplex algorithm.

The histogram of changes in the minimal value in Figure 9 shows that the differences are limited by $1 \cdot 10^{-4}$ for 90 of 136 cases. Additionally,

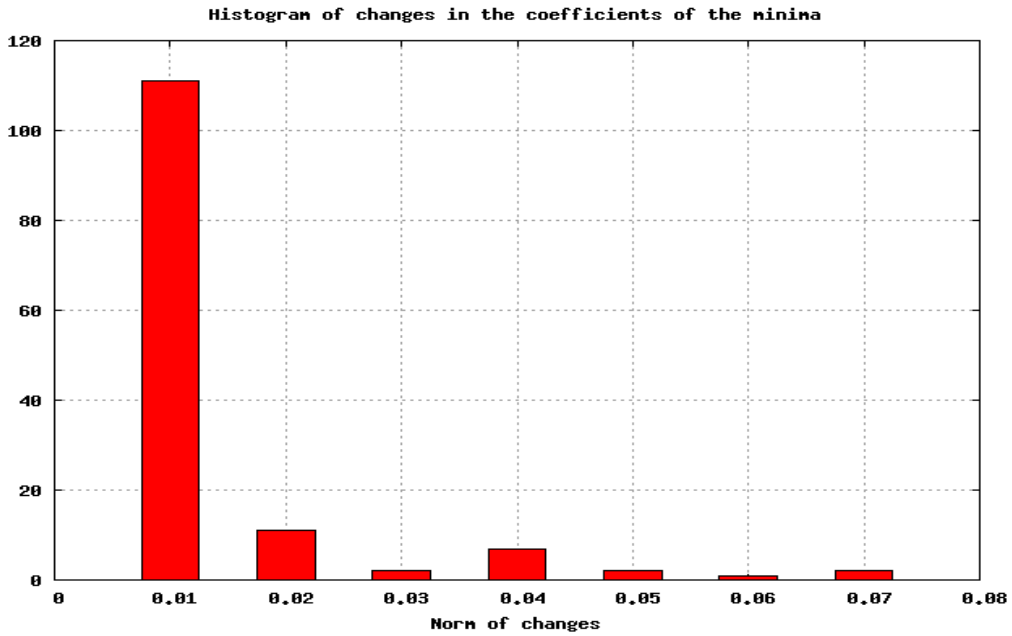

Figure 10: Histogram of changes in the coefficients of minima for different initial values in the Downhill Simplex algorithm.

more than 85% of the cases do not exceed the squared difference of $2 \cdot 10^{-4}$. Figure 10 shows that111 of 136 (82%) tests with varied initial values produce differences in the coefficients of less than 0.01. Furthermore, there are no cases producing differences of at most larger than 0.07. These results imply that the Downhill Simplex algorithm applied to the calibration is stable with respect to changes in the initial values. In addition to the quality and the stability of the algorithm, the convergence of the method is one of the most important evaluation criteria. The used algorithm converged for every set of market data and arbitrary initial values. To be more precise, it has taken $3,500$ iterations in average. Assuming the mentioned time for a function valuation of 0.1 seconds, the algorithm converges in 5 min. at the latest to an appropriate minimum. Summarizing all results, the algorithm produces adequate results in a reasonable time and is robust with respect to changes in the initial values.

5.2.4 Simulated Annealing

5.2.4.1 Description

The Simulated Annealing method is a (global) optimization method deliver-

ing remarkable results if a desired global extremum is hidden among many local optima. Unfortunately it does not guarantee the convergence to the global optimum, but especially combinatorial minimizations like the 'Traveling Salesman Problem' were solved efficiently by this technique. The method of Simulated Annealing is inspired by thermodynamics, especially the way of freezing and crystallizing liquids. In nature, the thermal mobility of atoms decreases for cooling liquids and ends in crystals forming the state of minimum energy for this system. The method of Simulated Annealing pics up this idea and allows more movements even uphill (worse solution) for 'high temperature'. Thus it is possible that the algorithm leaves local minima or valleys and increases the probability of finding the global extremum. The Boltzmann probability distribution controls the likelihood of exploring the space and allowing temporally worse movements. The probability is linked to the 'temperature' and the algorithm converges in dependence to a predefined annealing schedule. Empirical tests have shown that the temperature should be reduced sufficiently slowly to achieve the best results [Pre02].

In addition to the annealing schedule, there must be a procedure for iterating the solution. Following the ideas of [Pre02] this generator should be robust and stay efficient even in narrow valleys. Consequently we based the implementation of the Simulated Annealing method on a modified Downhill Simplex algorithm. The modification allows uphill movements whose maximal length depends on a synthetic temperature T controlled in the annealing schedule. In the limit $T \to 0$, this algorithm reduces exactly to the Downhill Simplex method.

5.2.4.2 Results

In analogy to the tests of the Downhill Simplex algorithm, we applied the Simulated Annealing method to the same market data. As previously described, the algorithm makes use of the Downhill Simplex method. According to the results in section 5.2.3.2 we used parameter set 1. We applied the calibration method to a series of market data starting in Jan. 2008 and ending in Jan. 2010. Figure 11 presents the results of these calibrations to at-the-money swaptions in form of the minimal values according to equation (25) and compares them to the corresponding outcomes of the Downhill Simplex algorithm. We used the same set of swaptions as described in section 5.2.3.

The minimal values of the calibration in the time series vary between

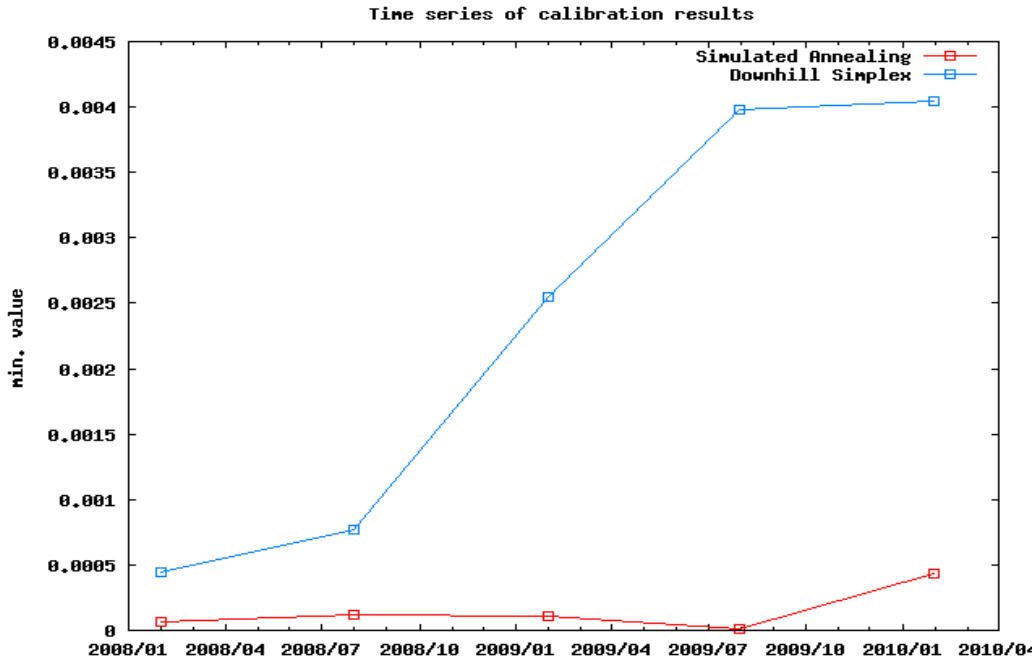

Figure 11: Time series of minimal values of the calibration to swaptions based on the Simulated Annealing algorithm and comparison to the Downhill Simplex algorithm.

$1.1 \cdot 10^{-5}$ in July 09 and $4.3 \cdot 10^{-4}$ in Jan. 10. The level of minimal values measuring the quality of the calibration is significantly lower in comparison to the results of the Downhill Simplex algorithm. And the check of the calibration quality according to 4.4 shows that it fulfills the highest standards in any case.

In addition to the level of minimal values, the range is much tighter although the level of corresponding swaption volatilities enlarges in times of financial crisis. As usual, we analyzed the sensibility of the optimization method with respect to the initial values. Therefore, we varied the starting points in a fixed scenario (parameter, market data, ...) and compared the position of the minima according to the vector norm (28). In total, we applied 171 different initial values. The differences in the minimal values and in the coefficients of the minima are presented in Figure 12 and Figure 13 .

The histogram of changes in the minimal value in Figure 12 shows that 136 of 171 (80%) cases end up in minimal values whose squared differences do not exceed $2 \cdot 10^{-4}$. Figure 13 shows that130 of 171 (76%) of initial values

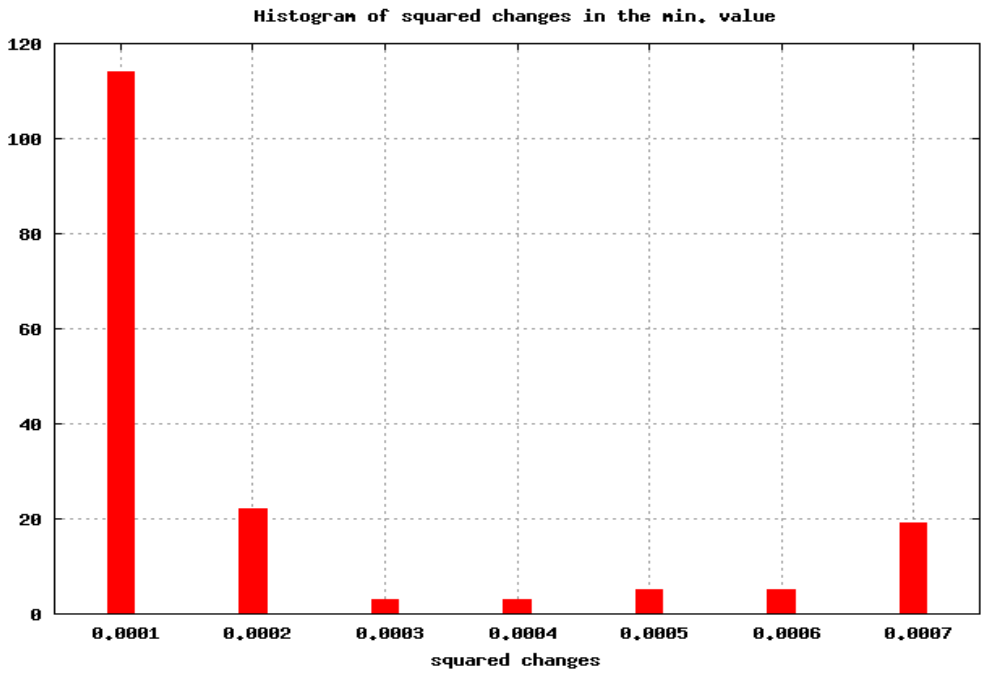

Figure 12: Histogram of changes in the minimal values for different initial values in the Simulated Annealing algorithm.

end up in a minimum whose coefficients do not vary more than $2.5 \cdot 10^{-3}$ with respect to the vector norm. The test generated furthermore some outcomes with quite large differences i.e. at least 0.2. The number of these alternative minima is small (27 of 171) and the minimal values do not enlarge. The setup of the calibration problem does not imply a unique (global) minimum and it might happen that there are several (local) minima with values close to the global one. But, nevertheless, the algorithm does not appear as stable as the Downhill Simplex algorithm concerning the position of the minimal values. Due to the operating mode, the Simulated Annealing method explores the optimization space much further. Thus it is not surprising, that the algorithm detects other and better minima. In the test, only the coefficients of the minimum vary slightly, but the minimal values are significantly better than the outcomes of the Downhill Simplex Algorithm.

The Simulated Annealing method and its behavior depend on the assumed annealing schedule. We tested several types like linear and exponential cooling schemes and some adaptive schemes. It turned out, that the linear cooling scheme delivers the best results, if the cooling factor is chosen sufficiently small. In addition, one can change the starting temperature to

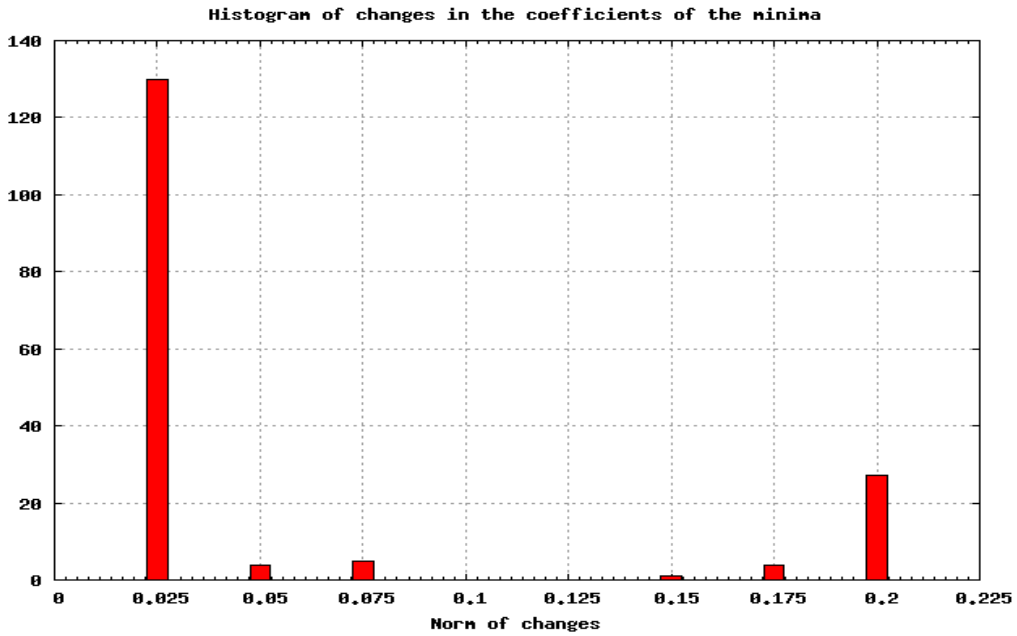

Figure 13: Histogram of changes in the coefficients of minima for different initial values in the Simulated Annealing algorithm with respect to the norm defined in equation (28).

determine the maximal probability of accepting worse results and possibly leaving local minima. The initial temperature and the cooling factor in the used scheme determine the number of iterations in the algorithm. Assuming a fixed cooling factor, one should assign the initial temperature as small as possible by ending up in a reasonable minimum. The tests have shown that an initial temperature of 5 is sufficiently large. Consequently, we used this setup for all calibrations based on Simulated Annealing.

Summarizing the results, one can state that the Simulated Annealing algorithm delivers good results and it is stable with respect to changes in the initial values. Furthermore, one can choose a small starting temperature which implies a small number of iterations.

5.2.5 Genetic Optimization

5.2.5.1 Description

The method of Genetic Optimization is a stochastic optimization method which incorporates probabilistic elements [Pol08] and uses some analogies

to the evolution theory in biology developed by Charles Darwin. Based on a randomly chosen set of potential solutions, the algorithm tries to find a global optimum by creating new solutions. This is done by combining and adjusting the previous solutions stochastically. The iterations within the algorithm are determined by three operators: selection, cross and mutation. The application is based on the binary code of the possible solutions, i.e. a string of 0/1. Therefore each coefficient is encoded to a binary representation consisting of single bits and all these codes are combined to a string representing one possible solution. First the cross operator selects a pair of solutions x and y with a predefined probability. The corresponding binary codes are both cut in the middle (after half of the number of bits) and one reaches the representation $x = x_1 \cup x_2$ and $y = y_1 \cup y_2$. As a second step, the cross operator swaps the ends of both binary codes and creates two new solutions $\overline{x} = x_1 \cup y_2$ and $\overline{y} = y_1 \cup x_2$. The mutation operator is applied to a single solution respectively its binary code. Each bit is changed with a given probability and consequently new solutions are created stochastically. As a last step, the selection operator identifies the best/worst solutions so far and duplicates the best by deleting the worst at the same time. The selection operator implies a concentration of the set of solutions somewhere in the optimization space. The algorithm stops, if a solution is found, that is sufficiently close to the optimum or the set of solution does not vary any longer.

As the genetic optimization method is based on some stochastic part included in the creation of the initial set of solutions and in the continuous adjustment, the detection of the global minimum is not guaranteed. The convergence is controlled by the set of initial values and typically we have to use a large set of numbers to explore the space of solutions. Consider for example a 9-dimensional optimization problem with coefficients that have values in an interval of length 1. If we discretize each interval with a step size of 10^{-3}, the resulting discrete solution space includes 10^{24} possible values. The discretization is incorporated by the choice of the binary code specially the number of used bits. So, we need a sufficient large set of candidates to reach adequate results with high probability. As the valuation of the function to minimize is expensive, the optimization may take long time to find the optimum. This effect is even intensified by the fact, that sometimes solutions are reevaluated at subsequent steps, since they could be generated several times.

The Genetic Optimization method searches the (global) optimum just in a discrete and finite space. The discrete space is created by discretization

of the continuous one and hence it is an approximation. The discretization controls the quality of the approximation and in the limit 'grid size $\to 0$' these spaces are equivalent. Thus, the optima of the discrete and the continuous space correspond in the limit as well.

5.2.5.2 Results

The Genetic Optimization method does not guarantee the convergence to the global minimum. The behavior of the algorithm is mainly determined by some specifications like the number of candidates, the probabilities of cross and mutation and the discretization of the domain of each coefficient. The choice of the probabilities is mainly independent of the problem. These values affect the exploration of the optimization space by the algorithm. In contrast the number of candidates and the discretization are strongly problem-specific. The range and the grid size of the discretization depends on the possible values of the coefficients and on the sensibility with respect to small changes. The minimization function for the calibration shows strong sensitivity with respect to changes in some coefficients like the constant shift. Thus the grid size has to be chosen quite small to represent all possible values accurately. If the grid size is chosen too large, the constructed discrete space is not a good approximation to the continuous one and consequently the results would not contain any conclusions. As the coefficients of the volatility parametrization effect the minimization function differently, the choice of the grid size should be done separately for each component. One should implement equidistant discretizations in each dimension to simplify the valuation. The number of candidates depends on the dimension of the problem and on the used discretization. The elements of the initial set are the origin of the exploration and even if the evolution is smart, the number of candidates should be chosen sufficiently high. Within the mentioned example, there are 10^{24} possible solutions and one should at least use $10,000$ candidates to cover a significant part. If the number is chosen too small, then the exploration is limited because of the selection operator. This operator forces a concentration somewhere in the space by duplication of the best solutions. If there are not many solutions outside this area left, then the space will not be well explored with high probability.

Unfortunately the valuation of the minimization function is time consuming. Within the mentioned example with 10^{24} possible values, one should at least use $10,000$ candidates. Numerical tests have shown, that the algorithm needs in average at least 500 iterations to identify the global optimum. The

function valuation takes in average 0.1 seconds and this implies a total computation time of about 140 hours. In order to decrease the computation time, one has to decrease the dimension or to coarsen the grid, but this implies worse results and with high probability inadequate ones. Summarizing, this optimization method can only be applied reasonably to low dimensional calibration problems.

5.3 Conclusions

The presentation of the results produced by the different minimization algorithms has shown that the Downhill Simplex algorithm delivers reliable and reasonable results. Furthermore this methods is robust with respect to changes in the initial values and thus it can reasonably be applied to the calibration problem. Nevertheless the Simulated Annealing algorithm seems to produce slightly better results, because it incorporates the possibility to leave local minima with a predefined probability. Thus the probability of detecting the global minimum is higher in comparison to the Downhill Simplex algorithm which is originally a local minimization algorithm. One possible extension not discussed so far is the combination of both. First one applies the Simulated Annealing algorithm with a high initial temperature to explore the space. Afterwards one starts the Downhill Simplex algorithm with the results of the Simulated Annealing method as initial values. Numerical tests have shown that this combination improves the results slightly, but there was no substantial gain measurable. This is not really surprising, because the Simulated Annealing algorithm is based on the Downhill Simplex method and in the limit $Temperature \to 0$ both methods are identical. The subsequent application just enlarges the number of iterations.

The Newton algorithm does not produce stable results as it makes use of derivations. But in combination with a previous Simulated Annealing minimization, it just has to work locally in a normal area. Unfortunately the algorithm sometimes runs far away from the initial values which destroys this method again. In the successful case the results are not superior to the corresponding ones produced by the Downhill Simplex algorithm.

The Genetic Optimization algorithm is really auspicious, because it explores the space extensively. Unfortunately the space in this setup is often of high dimension. In combination of a function whose valuation takes quite long, the optimization method is not practicable. The Powell algorithm offers a good opportunity of dimensional reduction. Unfortunately the minimization function does not offer any beneficial features, thus each one dimensional

optimization is hard to solve globally. This fact destroys the produced advantages of decomposing the dimensions. Consequently the Powell algorithm does not work convincingly in the setup of the calibration of the Cheyette Model.

Summarizing, the minimization methods Downhill Simplex and Simulated Annealing deliver the best and reliable results for the calibration of the Cheyette model. In addition to the analysis of the behavior and the accuracy of the algorithm we present the results of the calibration in form of the volatility surface. Using the volatility parametrization (27) and calibrate the model to 16 swaptions observed at Jan. 2008 (red) and July 2009 (green) we receive two volatility surfaces presented in Figure 14 . The results of the calibration as a comparison between the Cheyette model and the Black-Scholes model in terms of implied volatility is given in Table 2 and Table 3 . The calibration to market data observed at the end of Jan. 2008 fulfills all conditions on the quality defined in section 4.4 and consequently it is evaluated as 'good'. The results of the second calibration violates one secondary condition and thus it is evaluated as 'passed'. The minimal values are given by $6.5 \cdot 10^{-3}$ (Jan. 08) and $8.5 \cdot 10^{-3}$ (July 09). The implied surfaces have different shapes and includes some negative values as well. This effect has happened, because the calibration is performed on a market fraction. The extrapolation of these results might imply some negative values. The volatility surface of Jan. 08 shows several waves, while the surface of July 09 appears quite calm. This example demonstrates that the appearance of the volatility surface changes over time and the model should support maximal flexibility by the choice of the volatility parametrization.

In addition to the calibration to swaptions, we have applied the calibration to caps and floors as well. Again we have used the market data of Jan. 08 (red) and July 09 (green) and the implied surfaces are displayed in Figure 15 . The calibration to caps/floors is evaluated as 'passed' in both cases and the minimal value is given by $3.8 \cdot 10^{-5}$ (Jan. 08) and $1.1 \cdot 10^{-4}$ (July 09). The implied volatility surfaces have different shapes again, but in this case the volatility surface of Jan. 08 appears quite regular and calm. This appearance differs from the structure of the volatility implied by the calibration to swaptions and underlines the need of flexible models.

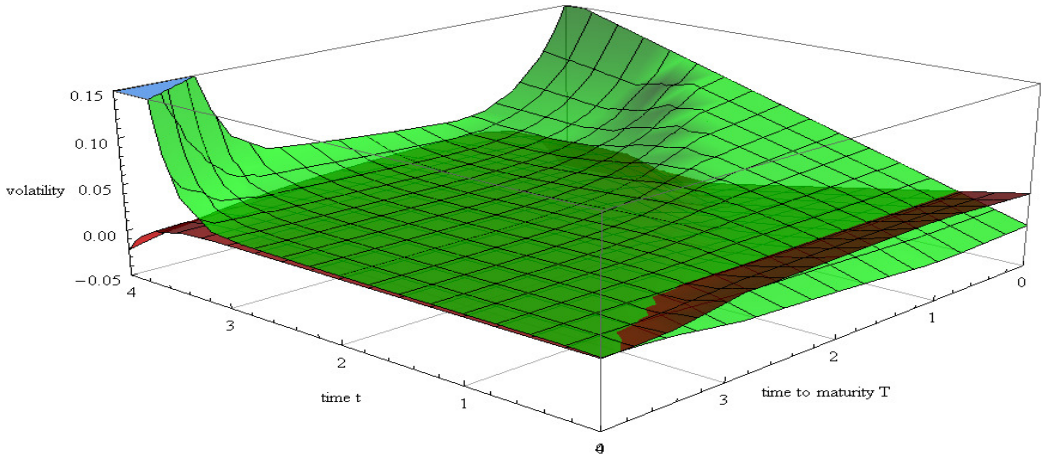

Figure 14: Volatility surface implied by the calibration with the Simulated Annealing method to swaptions with market data observed at the end of Jan. 2008 and July 2009.

Option lifetime	Swap lifetime	Cheyette volatility	Black-Scholes volatility	Residual
0.25	2.0	38.45 %	41.40 %	-0.0294
0.25	3.0	39.62 %	36.10 %	0.0352
0.25	4.0	33.44 %	33.00 %	0.0044
0.25	5.0	28.71 %	31.00 %	-0.0229
0.25	6.0	24.35 %	29.70 %	-0.0534
1.0	2.0	35.99 %	33.70 %	0.0229
1.0	3.0	31.70 %	29.90 %	0.0180
1.0	4.0	28.69 %	27.90 %	0.0079
1.0	5.0	26.03 %	26.50 %	-0.0046
1.0	6.0	24.30 %	25.50 %	-0.0119
4.0	1.0	22.07 %	19.40 %	0.0267
4.0	2.0	15.93 %	18.30 %	-0.0236
4.0	3.0	17.71 %	17.50 %	0.0021
4.0	4.0	18.34 %	17.30 %	0.0021
4.0	5.0	18.58 %	17.20 %	0.0138
4.0	6.0	18.99 %	17.10 %	0.0189

Table 2: Results of the calibration to swaptions with market data observed at the end of January 2008. The comparison between the Cheyette model and Black-Scholes model is done in terms of implied Black-Scholes volatilities.

Option lifetime	Swap lifetime	Black-Scholes volatility	Cheyette volatility	Residual
0.25	2.0	14.20 %	12.90 %	0.0130
0.25	3.0	14.10 %	13.80 %	0.0031
0.25	4.0	14.11 %	14.40 %	-0.0029
0.25	5.0	14.10 %	14.50 %	-0.0039
0.25	6.0	14.11 %	14.40 %	-0.0029
1.0	2.0	14.76 %	15.10 %	-0.0033
1.0	3.0	15.27 %	15.30 %	-0.0002
1.0	4.0	15.51 %	15.40 %	0.0011
1.0	5.0	15.95 %	15.60 %	0.0035
1.0	6.0	16.27 %	15.50 %	0.0077
4.0	1.0	16.52 %	16.70 %	-0.0017
4.0	2.0	16.09 %	16.60 %	-0.0050
4.0	3.0	16.07 %	16.40 %	-0.0032
4.0	4.0	15.90 %	16.20 %	-0.0029
4.0	5.0	15.75 %	15.80 %	-0.0004
4.0	6.0	15.63 %	15.50 %	0.0014

Table 3: Results of the calibration to swaptions with market data observed at the end of July 2009. The comparison between the Cheyette model and Black-Scholes model is done in terms of implied Black-Scholes volatilities.

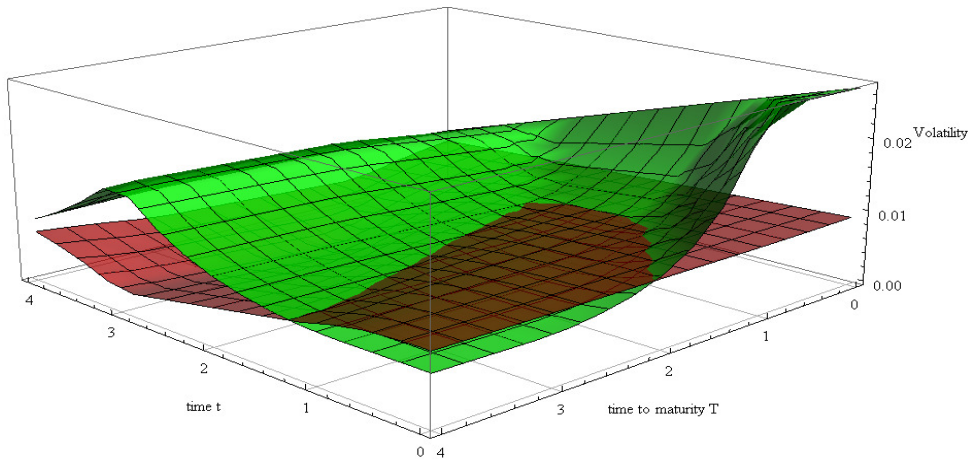

Figure 15: Volatility surface implied by the calibration with the Simulated Annealing method to caps/floors with market data observed at the end of Jan. 2008 and July 2009

A Appendix

The calibration is based on implied volatilities quoted at the market for swaptions and caps/floors. In the following, we will present the used market data published by 'Moosmüller & Knauf AG'.

	Option Lifetime									
Swap Lifetime	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{6}{12}$	$\frac{9}{12}$	1	1.5	2	3	4
1	24.0	22.9	22.1	21.7	21.6	21.4	19.9	18.9	17.4	16
2	26.4	24.9	23.5	21.8	21.0	20.3	19.0	18.2	16.7	15.4
3	26.2	24.5	23.0	21.1	20.1	19.4	18.2	17.4	16.1	14.9
4	25.5	23.7	22.3	20.4	19.3	18.6	17.6	16.8	15.5	14.4
5	24.7	22.7	21.4	19.6	18.6	17.8	16.9	16.2	15.0	13.9
6	23.3	21.5	20.4	18.7	17.8	17.2	16.3	15.6	14.4	13.5
7	21.9	20.3	19.4	17.9	17.1	16.5	15.7	15.0	14.0	13.2
8	20.5	19.2	18.4	17.0	16.4	15.8	15.1	14.5	13.6	12.9
9	19.3	18.2	17.6	16.4	15.7	15.3	14.6	14.1	13.3	12.7
10	18.1	17.3	16.7	15.7	15.1	14.8	14.2	13.8	13.1	12.5

Table 4: Swaption volatility matrix observed at the end of January 2008. The volatilities are quoted in percentage and the lifetimes are quoted in years.

	Option Lifetime									
Swap Lifetime	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{6}{12}$	$\frac{9}{12}$	1	1.5	2	3	4
1	21.5	21.6	21.5	20.9	20.9	20.9	20.0	19.0	17.8	17.1
2	24.1	23.5	23.0	21.9	21.0	20.4	19.1	18.0	17.3	16.6
3	23.3	22.7	22.2	21.3	20.4	19.8	18.6	17.6	16.9	16.2
4	22.4	21.8	21.3	20.3	19.6	18.9	17.8	17.1	16.5	15.7
5	21.4	20.9	20.3	19.3	18.6	17.9	17.2	16.6	15.9	15.1
6	20.7	19.7	19.0	18.2	17.6	17.0	16.4	16.0	15.4	14.6
7	19.9	18.7	18.0	17.3	16.8	16.4	16.0	15.7	15.0	14.2
8	19.0	18.0	17.3	16.7	16.4	16.1	15.7	15.4	14.6	14.0
9	18.1	17.3	16.7	16.2	16.1	15.8	15.5	15.1	14.4	13.7
10	17.2	16.6	16.2	15.9	15.8	15.6	15.3	14.9	14.2	13.6

Table 5: Swaption volatility matrix observed at the end of July 2008. The volatilities are quoted in percentage and the lifetimes are quoted in years.

	Option Lifetime									
Swap Lifetime	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{6}{12}$	$\frac{9}{12}$	1	1.5	2	3	4
1	72.2	63.8	59.4	51.9	45.9	40.3	31.8	26.1	21.1	18.9
2	65.7	58.9	53.0	43.5	38.1	33.9	27.9	24.0	20.3	18.5
3	59.6	53.5	48.4	39.6	34.6	31.2	26.5	23.3	20.2	18.5
4	55.3	50.0	45.5	37.3	32.8	29.7	25.5	22.7	20.1	18.7
5	52.3	47.5	43.6	35.7	31.4	28.6	24.9	22.3	20.1	18.8
6	49.8	45.5	42.0	34.4	30.2	27.8	24.4	22.2	20.2	18.7
7	48.1	44.1	40.9	33.7	29.5	27.2	24.0	22.2	20.1	18.7
8	46.6	43.0	40.0	33.0	29.0	26.9	23.8	22.1	20.2	18.8
9	45.1	41.8	39.0	32.5	28.5	26.6	23.9	22.3	20.4	19.0
10	44.5	40.8	38.4	32.1	28.2	26.5	24.0	22.6	20.5	19.1

Table 6: Swaption volatility matrix observed at the end of January 2009. The volatilities are quoted in percentage and the lifetimes are quoted in years.

	Option Lifetime									
Swap Lifetime	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{6}{12}$	$\frac{9}{12}$	1	1.5	2	3	4
1	54.6	52.0	50.5	47.8	45.2	42.2	32.8	27.7	22.2	19.4
2	44.4	42.9	41.4	38.2	35.9	33.7	28.4	24.8	20.6	18.3
3	38.2	37.4	36.1	33.6	31.7	29.9	26.4	23.4	19.7	17.5
4	34.6	34.0	33.0	31.1	29.6	27.9	24.8	22.3	19.0	17.3
5	32.3	31.8	31.0	29.4	28.1	26.5	23.9	21.5	18.8	17.2
6	30.4	30.3	29.7	28.1	26.9	25.5	23.2	21.2	18.7	17.1
7	29.1	29.1	28.6	27.3	26.2	25	22.9	20.9	18.7	17.0
8	28.3	28.4	28.0	26.8	25.9	24.7	22.7	20.9	18.6	17.0
9	27.6	27.8	27.5	26.5	25.6	24.6	22.6	20.8	18.6	17.0
10	27.0	27.4	27.1	26.3	25.5	24.4	22.5	20.8	18.7	17.1

Table 7: Swaption volatility matrix observed at the end of July 2009. The volatilities are quoted in percentage and the lifetimes are quoted in years.

	Option Lifetime									
Swap Lifetime	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{6}{12}$	$\frac{9}{12}$	1	1.5	2	3	4
1	45.6	45.9	46.0	47.7	48.3	47.0	39.6	34.8	27.8	23.1
2	40.3	39.8	39.3	38.9	38.0	37.0	32.7	29.5	24.0	20.4
3	33.5	33.2	33.6	33.4	33.0	32.0	28.8	26.2	22	19.1
4	28.7	28.8	29.6	29.7	29.1	28.2	25.9	24.0	20.7	18.3
5	25.8	26.0	26.3	26.8	26.4	25.6	23.8	22.5	19.9	17.9
6	23.8	24.1	24.1	24.5	24.3	23.8	22.6	21.5	19.3	17.6
7	22.2	22.7	22.5	23.1	23.0	22.7	21.7	20.8	18.8	17.5
8	21.1	21.5	21.4	22.0	22.1	21.9	21.0	20.3	18.5	17.4
9	20.1	20.6	20.5	21.3	21.4	21.2	20.5	19.8	18.4	17.3
10	19.3	19.7	19.9	20.5	20.7	20.7	20.0	19.3	18.2	17.3

Table 8: Swaption volatility matrix observed at the end of January 2010. The volatilities are quoted in percentage and the lifetimes are quoted in years.

	Dates				
Cap Lifetime	Jan. 2008	July 2008	Jan. 2009	July 2009	Jan. 2010
1	17.8	15.6	51.3	52.8	63.5
1.5	18.0	18.9	43.9	49.6	59.6
2	18.0	20.2	38.7	43.1	51.8
3	17.4	20.8	28.5	33.3	38.4
4	16.8	19.9	23.7	28.4	31.9
5	16.2	19.1	21.1	25.1	27.5
6	15.7	18.4	19.2	22.5	24.3
7	15.2	17.8	18.0	20.6	21.9
8	14.7	17.1	17.2	19.3	20.2
9	14.4	16.6	16.7	18.3	19.0
10	14.0	16.1	16.2	17.5	18.1

Table 9: Cap (ATM) volatility matrix observed at different times. The volatilities are quoted in percentage and the lifetimes are quoted in years.

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