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# FX Column: Can Vega of a Double-No-Touch be Positive?

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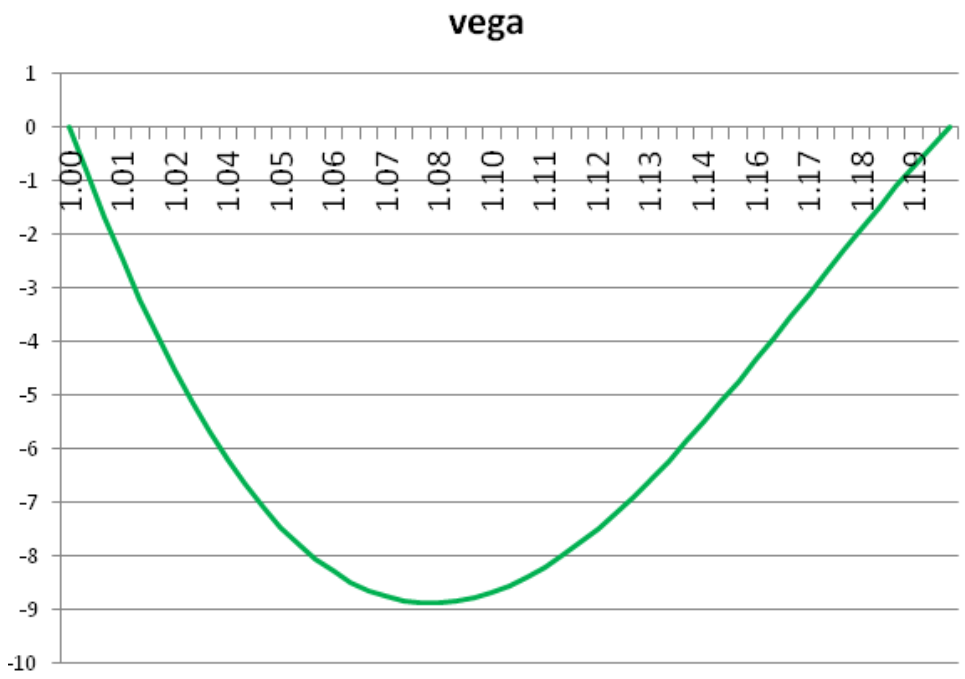
While teaching classes on FX exotics over the last 20 years, there isn't one day I don't learn something new. In today's example I would like to share one of the things I learned about the vega of a double-no-touch (DNT).

**Product definition** The DNT pays a fixed amount of currency at maturity if the FX spot trades inside a pre-specified range between inception of the trade and maturity. The range is often chosen symmetrically around initial spot.

**Vega with one valley**

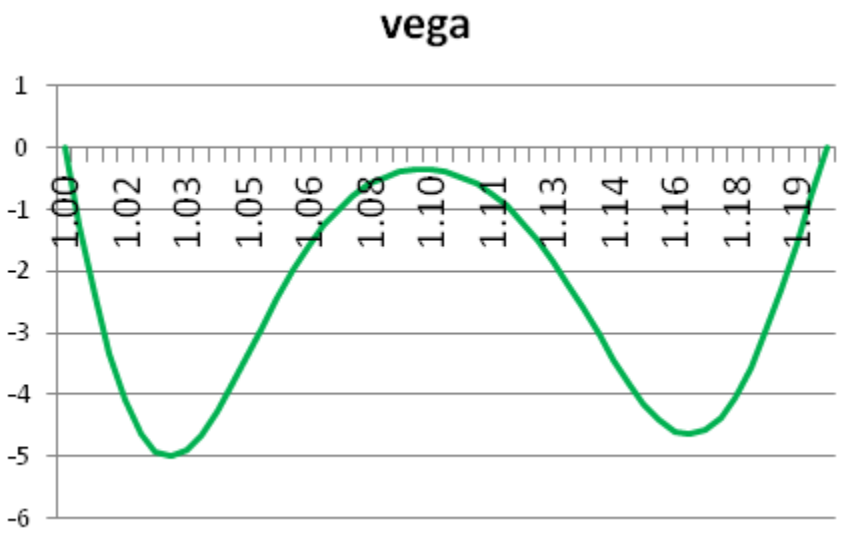
Buying a DNT will typically generate a short vega position, which means that if volatility goes up, then the value of the DNT goes down. This is quite intuitive: if volatility goes up, then the probability of the spot hitting any of the two barriers increases; therefore, the chance of getting a payment will be lower. This is illustrated by the following graph showing vega of a DNT[1] on the spot space. And it is the most common answer I get when I assign the task to plot the vega profile of a (long) DNT.




[1] Contract data: range 1.0000 – 1.2000 EUR-USD, 6M, payment in USD; market data: spot ref 1.1000, USD rate 2.00%, EUR rate -0.50%, ATM volatility 10%, no smile.

**Vega with two valleys**

Alternative plots look like a negative vega with two valleys as in the next graph. This is the second most common response.

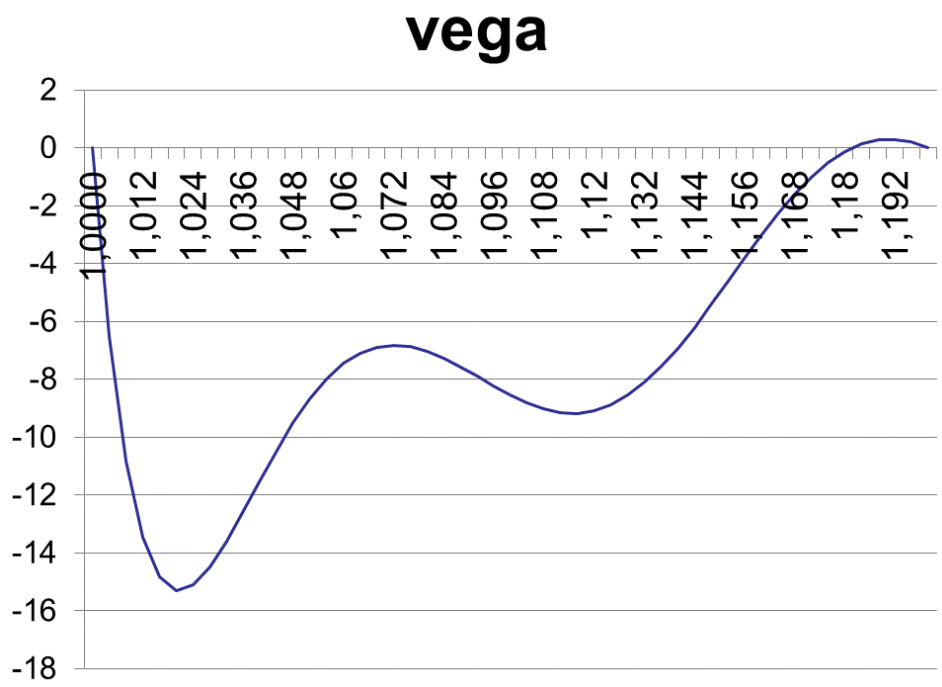



**Which one is right?**

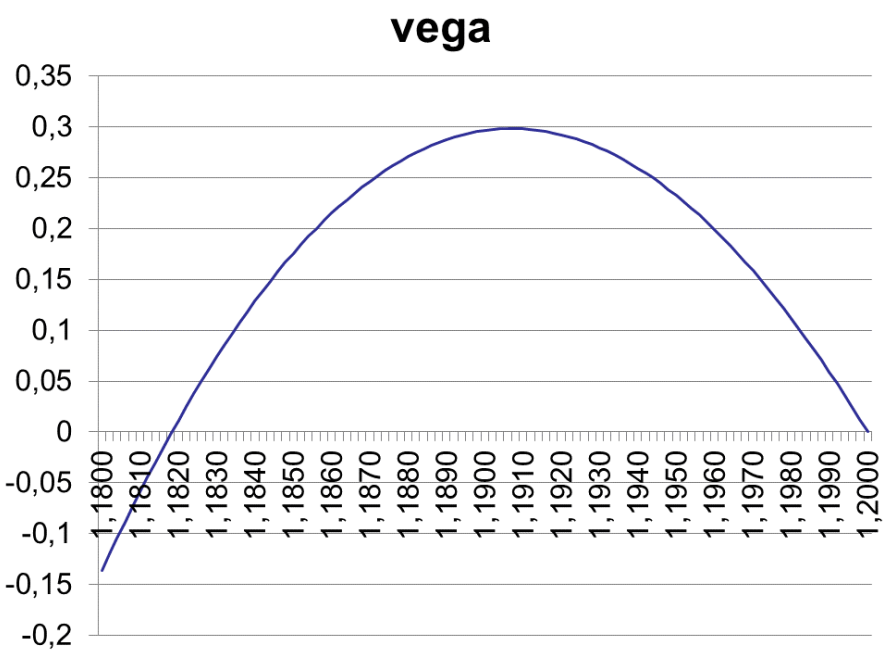
Generally, vega must be zero on the barriers, which is where the value of the DNT is zero and terminates once the spot hits any of the barriers. Whether it has two minima or just one depends on the variance, i.e. the product of volatility squared times the remaining time to go. The first graph of vega with one valley has a higher variance, in our example 6 months time to maturity; the second graph with the two valleys has a smaller variance, in our example 1 month time to maturity. The shape can also be produced by a very low volatility. In fact, there are always two valleys, but with higher overall variance, the valleys merge and look line one valley. What happens at spot 1.1000 with very little time left is that a higher volatility will not increase the probability of the spot hitting any of the barriers because they are both already too far away. The volatility sensitivity is concentrated near the barriers. With very little time left and a spot near one of the barriers, an increase in volatility will increase the hitting probability and therefore lower the value of the DNT.

When we look at the vega profile of a (long) DNT on the spot space, we easily verify by visual inspection that it is indeed negative, either with one or with two valleys. So far so good. So when I ask if vega of a DNT can be positive, then one of the common answers I get is that it can be if I go short a DNT. This is obviously not what I had in mind

In the next example I consider the same 1-year DNT in EUR-USD with range 1.0000 to 1.2000 and an initial spot of 1.1000.




Looking at the chart carefully, we spot a slightly positive vega near the 1.2000 barrier. When I first spotted this, I found it quite disturbing, and initially I thought it must be one of these features of the new Excel version. But, after zooming in to the range 1.1800 to 1.2000, I realized that vega on the spot space did indeed appear positive.




I double checked my code, verified the formula with finite differences and could not find a mistake. So what was going on?

**Explanation**

The example has been created using different market data. Taking a USD interest rate of 5% and EUR interest rate of 1.5%, we generate a *forward curve moving strongly up*, so starting from 1.1900 the forward rate reaches the barrier 1.2000 in one quarter. Hence, just looking at the forward curve, we can be sure the DNT will terminate early. This would happen if we didn't have volatility and the spot would just follow the forward curve. Consequently, with a very low volatility (4% in my example), the spot will stick to the forward curve very closely and lead to an almost sure early termination of the DNT and therefore a zero payoff. Then increasing the volatility will effectively increase the probability of the spot staying away from the upper barrier, in which case the vega profile is positive for spot levels between 1.1800 and 1.2000. In short, a higher uncertainty turns a sure death into a glimpse of hope.

And surely, you must have known this feature for a long time☺

**Applications of Double-Touch Contracts**

A double-no-touch is a very liquid instrument. Going long a DNT reflects a view of spot staying quiet over the lifetime of the contract. Long a DNT means typically short vega, so it is a contract one can buy and be vega short. Traders must be aware of the general intuition no being valid in market with strong forward curves. DNT and DOT serve as rebates to double knock-out and double-knock-in options. They also appear as building blocks in structured FX forwards and yield enhancing deposits like wedding cakes, towers and

onions. The general idea is to give the treasurer or investor a worst case effective FX rate or coupon – which happens if the DNT terminates early – and a best case, the best case occurring if the view of a quiet market turns out to be true.

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**References**

Stefan Ebenfeld, Matthias R. Mayr, and Jürgen Topper. An Analysis of Onion Deposits and Double-no-Touch Digitals. *Wilmott Magazine*, pages 68 – 77, September 2002.

Uwe Wystup, FX Options and Structured Products, 2nd Edition, Wiley, 2017

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