Reverse-Knock-Out Pricing Case Study: Stochastic Local Volatility vs. Vanna-Volga

Uwe Wystup - MathFinance

Today, let's revisit pricing a reverse knock-out option (RKO). This call or put option knocks out if at any time between trading time and the expiry date the spot hits or crosses a pre-specified barrier. For the RKO, the barrier is in the money, i.e. above the strike for a call, and below the strike for a put. With the barrier in the money, this option creates a large intrinsic value if the spot is near the barrier. This product is a popular building block in FX structured forwards for corporate treasurers, and also as part of Dual Currency Investments in the private banking industry. The buy-side would normally not trade stand-alone RKOs, unless they are hedge-funds deriving their adrenaline level from model arbitrage.

We consider a GBP put CHF call, strike 1.3708, down and out at 1.1389, expiring in 190 days on 8 Nov 2019, delivery date 10 Nov 2019, whose payoff is illustrated in Figure 1.

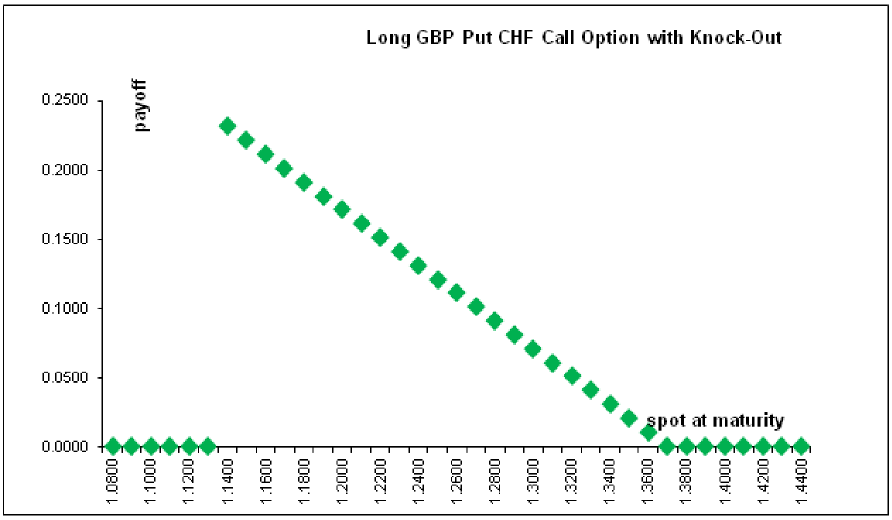

Figure 1: Down-and-out GBP put CHF call

We use the following market data on horizon 2 May 2019: spot reference 1.3309, 6M GBP deposit rate 0.7456%, CHF deposit rate -0.9155%, forward 1.3197, 9M GBP deposit rate 0.7648%, CHF deposit rate -0.948%, forward 1.3136, 6M ATM volatility 7.233%, 9M ATM volatility 7.385%. The volatility smile in Figure 2 shows a significant down skew (negative risk reversals), which reflects a market sentiment that Sterling is going to lose value against

Swissies, which makes hitting a lower barrier in GBP-CHF more likely. The smile curve also illustrates that the Heston model does not fit the input perfectly, as it is a parametric model with limited number of possible shapes, whereas the stochastic-local-volatility (SLV) models fit the smile well, as their local volatility component is designed in a way to well represent the vanilla market.

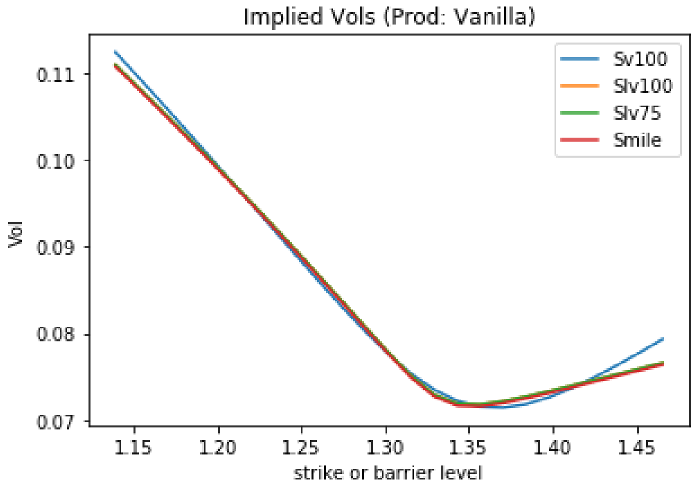

Figure 2: GBP/CHF volatility smile on the strike space. Smile refers to the MathFinance fit, Sv100 is the best possible fit in the Heston model, Slv100 is a stochastic-local volatility model with 100% stochastic mixing factor, Slv75 the corresponding one with 75% mixing factor.

Fitting the volatility smile gets worse if we try to use vanna-volga, see Figure 3. The vanna-volga fit does not naturally fit the smile at points other than ATM and 25-delta strikes. Consequently, when pricing and RKO with a barrier far away, in a vanna-volga approach, the price does not converge to the corresponding vanilla option price. This is a major drawback of vanna-volga: exotics are priced inconsistent with the vanilla options market. Additionally, we observe that even the way we design a vanna-volga model can lead to different smile curves (and hence to different exotics prices). Vv considers the cost of vanna when hedged with a risk reversal, and the cost of volga when hedged with a butterfly[1], whereas Vv2 considers the cost of vega, vanna and volga jointly hedged with three vanilla options.

[1] https://www.slideshare.net/wystup/fxmodels-slidesenglpublic

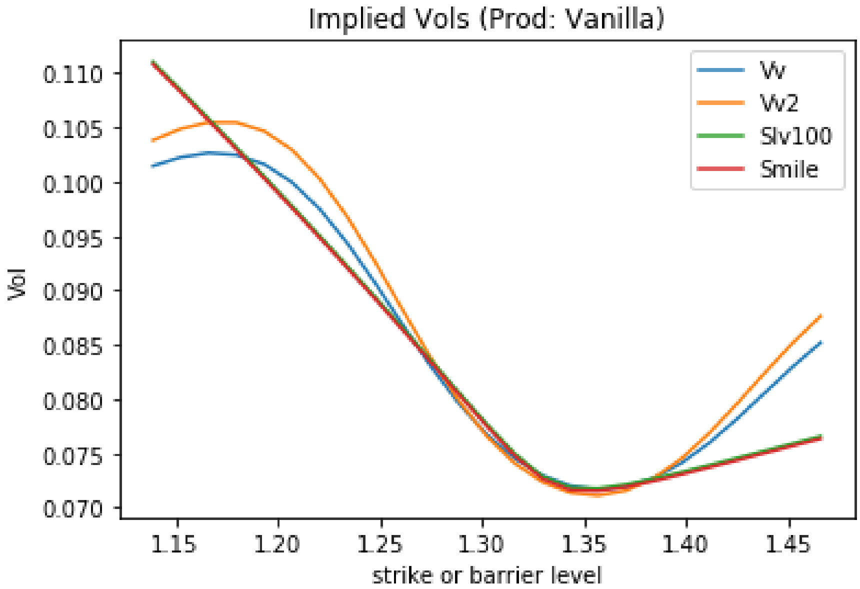

Figure 3: GBP/CHF volatility smile on the strike space. The two versions of vanna-volga Vv and Vv2 fit the smile only at the money and for 25-delta strikes.

Next let us turn to the pricing of the RKO in various models. Figure 4 shows that all the local- and stochastic and their mixture models lead to price deviations from TV (theoretical value = Black-Scholes model price using ATM volatility). Deviations converge to zero as the barrier converges to the current spot (right-hand side), when all prices are zero in all models. If the barrier moves far away from current spot (left-hand side), the RKO prices should converge to the corresponding vanilla prices (which they do except for the vanna-volga approaches).

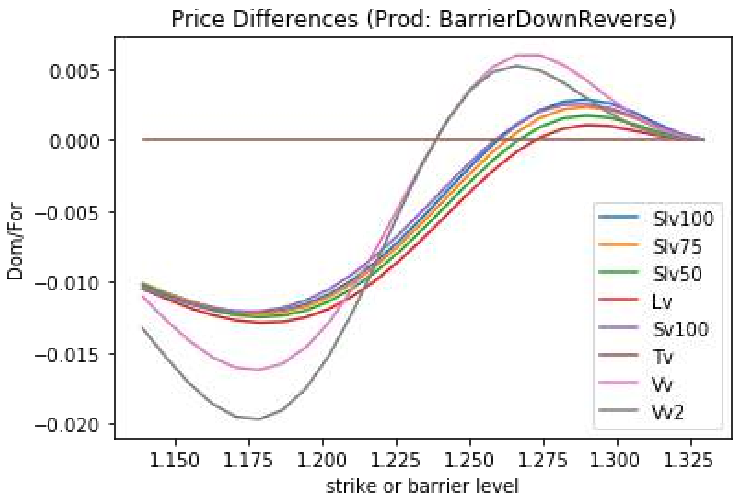

Figure 4: Differences to TV (theoretical value) of the GBP put CHF call RKO for a variety of models, measured in domestic currency per one unit of foreign currency notional. "Lv" represents a pure local volatility model. The barrier is moving so different option prices are calculated.

While the vanna-volga approaches still exhibit a broadly similar pattern, they still fall outside the typical range of local volatility and stochastic volatility pricing. Conclusion: vanna-volga based RKO pricing is at the most suitable for valuations for accounting and reporting purposes. For market makers, the prices are too far outside the SLV market standard, independent of the mixing factor. The vanna-volga fan club is still big though, because the computation of prices for a wide range of exotics is fast.

If a trader is used to seeing all derivatives portfolio's prices and Greeks in real time, vanna-volga is preferred for speed of calculation. An SLV model is preferred for

The markdown of 50-100 pips is significant. We illustrate now where it comes from. Recall that an RKO Put with strike K and barrier B can be statically replicated by a regular KO Call with strike K, a short strike-out call with K at B, and a no-touch at B with notional of the maximum intrinsic value K-B. We observe from Figure 5 that most the markdown is caused by the touch contract. If the price of the No-Touch is marked down, the price of the RKO is marked down, too.

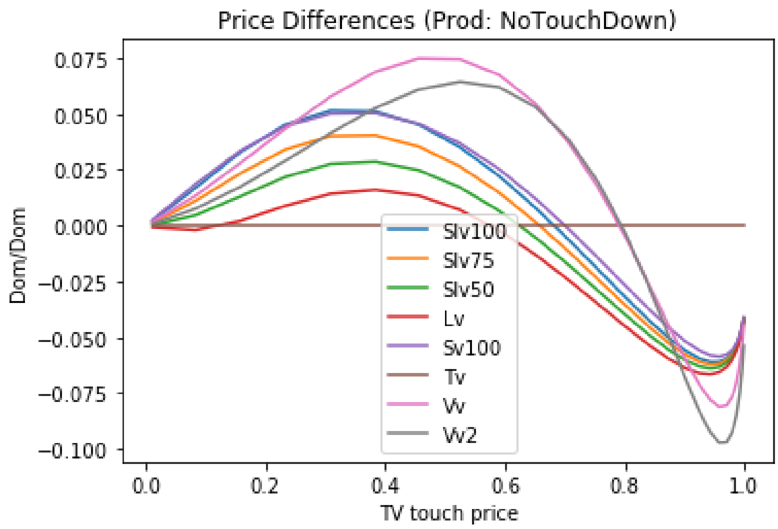

Figure 5: One-Touch mustache: differences to TV (theoretical value) of the CHF-paying No-Touch for a variety of models, measured in domestic currency per one unit of domestic currency notional. The lower barrier is moving so different prices are calculated.

For the sake of completeness, we illustrate the model impact for the RKO GBP call CHF put in Figure 6.

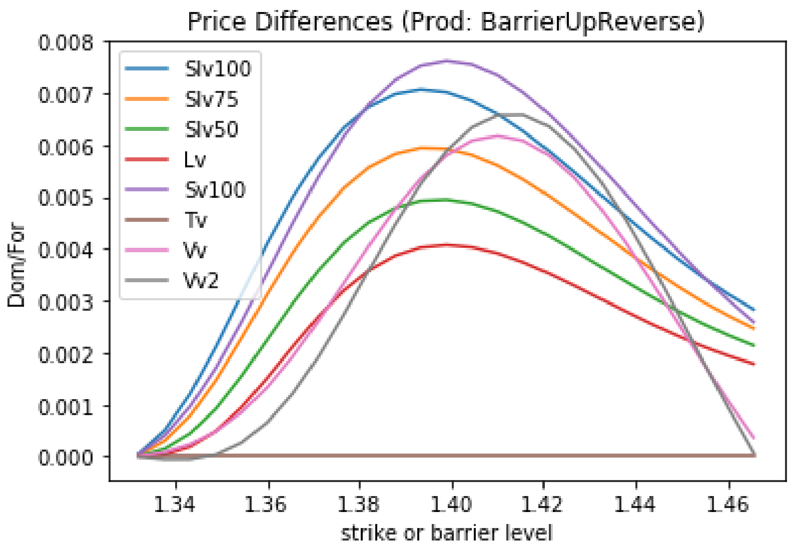

Figure 6: Differences to TV (theoretical value) of the GBP call CHF put RKO for a variety of models, measured in domestic currency per one unit of foreign currency notional.

Which of the mixing factors represents the market of traded RKO options can be determined by sliding the mustache between Lv (mixing factor 0%) and Slv100 (mixing factor 100%), so that most observable RKO prices are met. This way one could arrive at Slv75 or Slv50. Note that all RKOs are priced without any face-lifting. Face-lifting is used by many practitioners and means to replace the (discontinuous) payoff by a smoothed version. In case of the RKO, one does this with shadow barriers – further away barriers used only for pricing and hedging, or by replacing the boundary condition v(B)=0 by $Sv_S \leq \alpha v$, where v(S) is the value function of a RKO with spot at S and α is the maximum accepted leverage. Pricing the face-lifted version will obviously push the price of the RKO up, so at the end of the day, the mark down of 50-100 pips will be overshadowed by a face-lift adjustment.

The overall conclusion is that a sound application of SLV models leads to consistent pricing of exotics and vanilla options across the product and parameter range, a frequent requirement of model governance, - and a goal that vanna-volga based approaches will have a hard time fulfilling.

Frankfurt, 27 June 2019