FX Column: Vanna-Volga and the Greeks

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Many people ask us what is so special about calculating Greeks in the vanna-volga approach. Isn't it like calculating bumped values and ratios of differences?

On the other hand, did you ever wonder, why for instance most professional systems do not provide vanna-volga Greeks along with its "market price", when it is based on vanna-volga and other adjustments?

Before we dive into it, let us quickly recap the vanna-volga history:

First industrial use of the vanna-volga approach was put in place in the 1980s by O'Connor & Associates, a Chicago-based options trading firm, with emphasis on financial derivatives. The firm was acquired by Swiss Bank Corporation, which later merged with Union Bank of Switzerland to form UBS. Consequently, UBS used the model in the first online trading platform "UBS Trader" for first generation FX exotics. The approach was based on traders' rules of thumb. Publications on vanna-volga were scarce. The relevance of higher order Greeks vanna and volga (also called vomma) has been explained and summarized by Andrew Webb in [Webb, 1999].
This essay also mentions the need to estimate the first hitting time of a barrier. In the industry, SuperDerivatives holds a patent [Gershon, 2001]. Lipton and McGhee mentioned it 2002 [Lipton and McGhee, 2002]. An intuitive description of vanna-volga can be found in Wystup 2003 [Wystup, 2003], [Wystup, 2006]. A more mathematical vega-vanna-volga mainly in the context of volatility smile interpolation is by Castagna and Mercurio [Castagna and Mercurio, 2007], [Castagna and Mercurio, 2006], [Castagna, 2010]. Bossens et al. [Bossens et al., 2010] revised vanna-volga in 2010 in detail. Bloomberg documented its version of vanna-volga in 2007 [Fisher, 2007]. Nowadays, we can still find vanna-volga based pricing and valuation in most professional trading and risk-management systems. And this is where several issues arise.

pricing requires consistency risk management requires Greeks both challenging issues for vanna-volga approaches

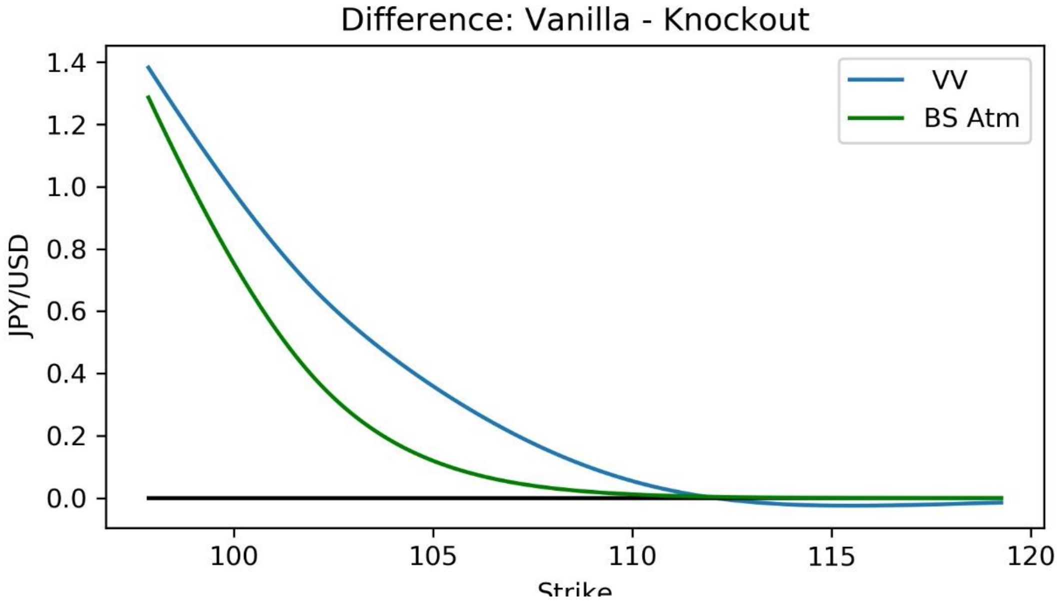

Figure 1: Difference of vanna-volga based KO call option value and its corresponding vanilla option value, strike on the x-axis

For exotics, we like to see prices consistent with vanilla options. A simple example is a barrier option, say a down-and-out call in USD-JPY with barrier 102, spot 109.24. When we move the barrier away, we want to observe a monotone convergence of the barrier option price to the vanilla option price. In particular, the barrier option price should never exceed the vanilla option price. Figure 1 shows the difference of the vanilla and the regular knock-out option value for varying strikes. In a Black-Scholes model with constant volatility the barrier option value is always below the vanilla option value.

Surprisingly we observe that in the vanna-volga approach the knockout option value can be slightly higher than the vanilla option value, so the difference of a vanilla minus knockout option value can become negative, see the blue curve in Figure 1. Analyzing this, it turns out that we are in a situation where for the vanilla option value the vanna-volga hedging costs are negative or, more generally, where the sum of the vanna and the volga term in the vanna-volga formula is negative and the vanna-volga value is below the flat volatility Black-Scholes value.

For knockout options the vanna and the volga term are usually multiplied with some factor smaller than one (often a function of the no-touch probability of the barrier) and the correction term to the Black-Scholes value is negative, but not negative enough anymore to cause a knock-out option value above the vanilla option value.

To force minimum consistency one can at the very least cap the barrier option value at the vanilla option value or floor their difference at zero, see Figure 2.

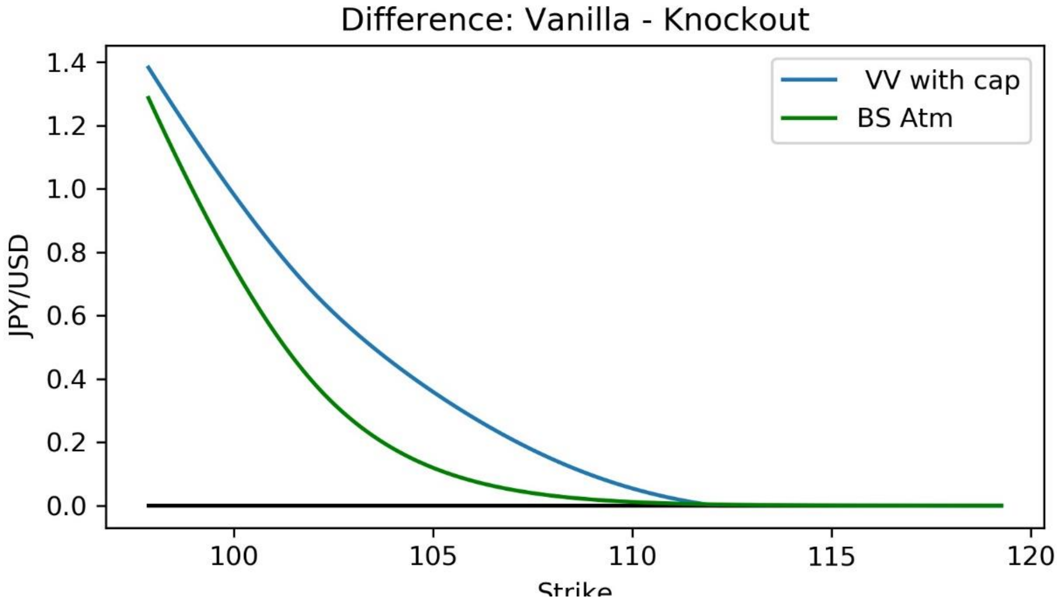

Figure 2: Difference of vanna-volga based KO call option value and its corresponding vanilla option value, floored at zero, strike on the x-axis

Implementing such a consistency rule is quite easy, but we now lose smoothness of the value function. And as we know this may cause jumps and spikes in the Greeks, especially when we compute derivatives by finite differences, i.e. bumping market data.

Figure 3 shows the vega profile for varying strikes for the above example computed by bumping volatility by an absolute value of 0.1%.

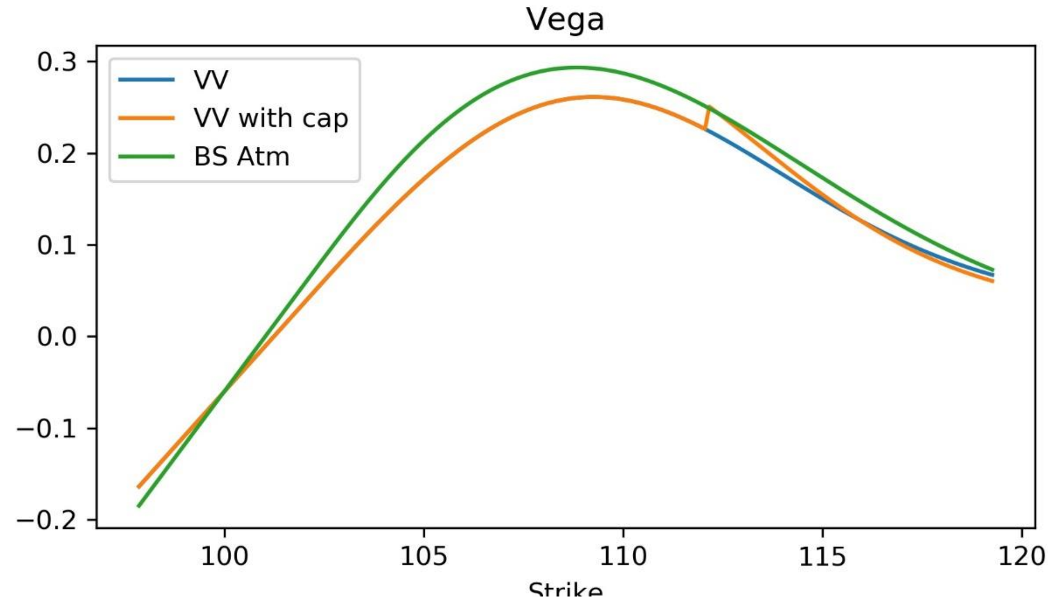

Figure 3: Vega on the strike space of a regular knock-out call, comparing vanna-volga approach with and without consistency rule (cap)

Figure 4 shows the gamma profile for varying strikes for the above example computed by bumping spot by an absolute value of 0.01.

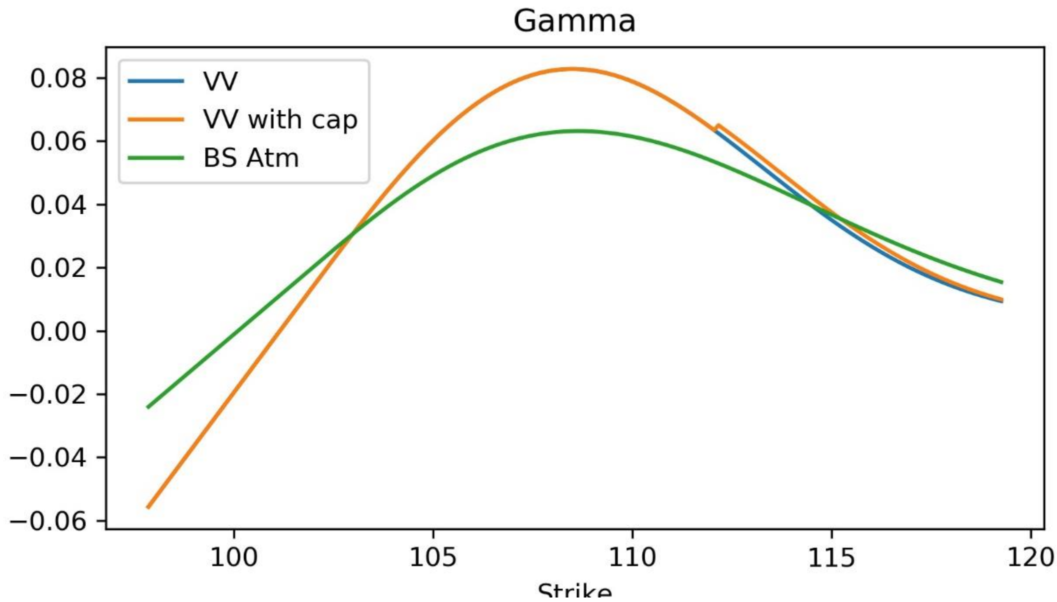

Figure 4: Gamma on the strike space of a regular knock-out call, comparing vanna-volga approach with and without consistency rule (cap)

In this example the kinks and jumps we see in the graphs are unpleasant, but not dramatic. The problem is that the kinks occur at parameter levels that are not easy to predict – in contrast to non-smooth behavior at a barrier level, which is known in advance and allows us to compute *one-sided* finite differences or shift the barrier.

Non-smooth value functions at non-predictable locations of kinks in combination with parameter bumping can now really lead to exploding Greeks, as can be seen in Figure 5. Just by selecting other strikes we hit a region where gamma takes an arbitrary value.

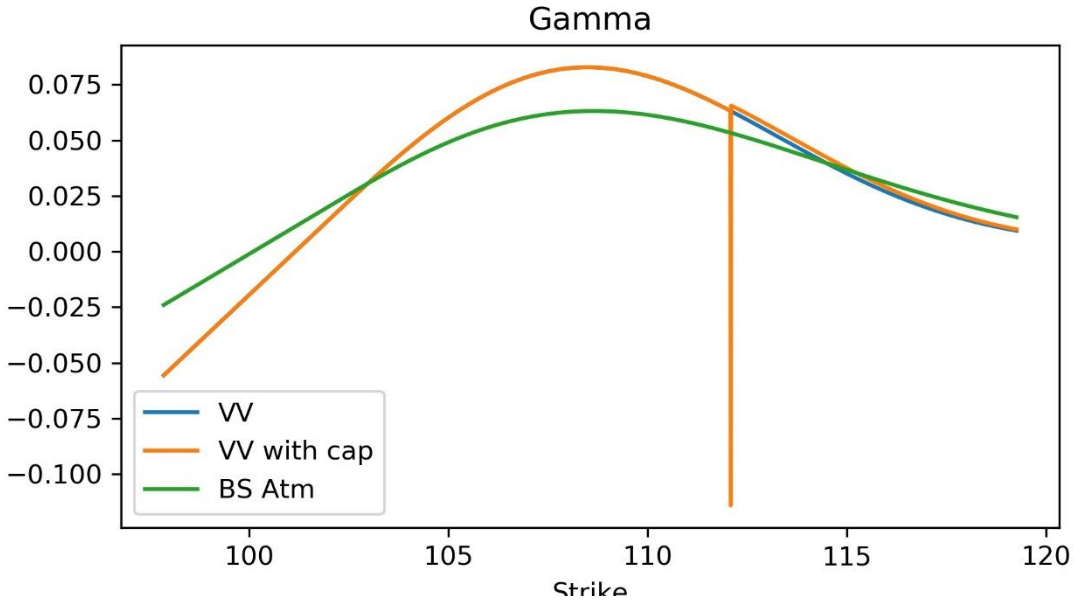

Figure 5: Exploding gamma on a different strike grid of a regular knock-out call, cause by a vanna-volga approach with consistency rule

Of course, one can try to model smoother transition rules between the curves for each consistency rule, but this will lead to an endless patchwork.

The situation becomes even worse if we value vanilla options from an already constructed smile surface. Then the vanilla options are valued correctly, but one would need to add many more rules to avoid inconsistencies caused by valuation based on the smile and valuation based on the vanna-volga approach [Wystup, 2019].

Valuing first generation exotics with the vanna-volga approach approximated their prices well in past and still yields very fast and simple rough indicative prices for touch contracts in FX markets. However, computing vanna-volga Greeks was and remains complicated. This is caused by having to impose many consistency rules to a seemingly simple vanna-volga formula.

Today we discussed only some of the technical difficulties and traps but didn't even explore if the Greeks computed in vanna-volga are useful and correct quantities for risk management.

Overall, we learn that if we impose conditions to make pricing of exotics in vanna-volga approaches consistent with vanilla options, we obtain instable Greeks. Without consistency conditions, the Greeks might be smoother, but the values may lead to arbitrage. It will require a lot of patchwork to cure all the headaches at one go.

Consequently, we consider it rather worth investing the time to compute stable Greeks within consistent models, knowing that there are still enough numerical challenges left. And with the model class of SLV and its variants we have models at hand, that are flexible enough to fit the prices in many FX option markets.

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