FX Column: The Pedigree of Exotics – Or Derivatives Lego

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How many exotic options or contracts are there actually? One answer could be that there are so many, and even within a contract there could be different variants, which makes them basically limitless. On the other hand, for a trader, the names of the contracts are not really important, but what matters are the risk features one needs to be aware when risk managing a book of exotics. Furthermore, exotics and vanilla options can be used a building block to duplicate others. A famous example is the reverse-knock-out barrier option (RKI), which can be replicated perfectly by a spread of regular knock-out barrier options and a no-touch contract. So, in counting exotics should we exclude all contracts that are replications of other building block?

First Generation Exotics:

The term *first generation exotic* does not refer to a clearly defined set of derivatives contracts, especially not in a legal sense. However, it is universally agreed that Foreign Exchange Transactions (spot and forward contracts) and vanilla options are not in the set. It is also universally agreed that flip-flop-kiko-tarns and correlation swaps are not in the set either. We can then classify first generation exotics by:

- **Time of Introduction:** Here we consider the history and the time when certain contracts first traded.
- **Existence of Standardized Deal Confirmations:** We would classify a transaction as first generation exotic if there exists a standardized deal confirmation template, such as the ones provided by ISDA.
- **Replicability:** We would classify a transaction as first-generation exotic if it can be statically or semi-statically replicated or approximated by spot, forward and vanilla option contracts.
- **Trading Volume:** We would classify a transaction as first-generation exotic, if its trading volume is sufficiently high (and the transaction is not a spot, forward or vanilla option).

There can also be other approaches to classify first generation exotics. I would like to point out that a first-generation exotic not necessarily needs to be a currency option. For example, a flexi forward can be considered a first-generation exotic in terms of both timing and standardization but is clearly not an option. A variance swap can be considered a first-generation exotic in terms of both standardization and replicability, but is clearly not an option, because there is no right to exercise. Classification by trading volume would change the set of first-generation exotics over time and is consequently not suitable for classification purposes. The various classifications would generate overlaps as well as differences. One could certainly argue to label barrier options as first-generation exotic, because they would satisfy all of the above: timing, standardization, replicability and volume. For Asian options, the timing criterion would make them first generation as they started trading in Tokyo in 1987, but there is – even in 2016 – no standardized deal confirmation provided by ISDA. Power options satisfy timing and replicability, but not standardization or trading volume. This leads to the effect that the transition between the generations is not strict and can depend on the person you ask and classification the respective person has in mind. A clean approach to classification could be sticking to the standardization, which would classify barrier options and touch products, as well as variance and volatility swaps as first-generation exotic, based on the existing ISDA Definitions and their supplements. The question which transaction is standardized

can then be viewed in light of ISDA's *Barrier Option Supplement* [1], which appeared in 2005. ISDA has extended the 1998 FX and Currency Option Definitions[2] to the range of touch products and single and double barrier options, including time windows for barriers. These are (a) options that knock in or out if the underlying hits a barrier (or one of two barriers) and (b) all kind of touch products: a one-touch [no-touch] pays a fixed amount of either USD or EUR if the spot ever [never] trades at or beyond the touch-level and zero otherwise. Double one-touch and no-touch contracts work the same way but have two barriers. The ISDA Barrier Option Supplement contains all the relevant definitions required to confirm these transactions by standardized short templates. It is clearly defined what a *barrier event* or a *determination agent* is. However, for purposes of classification, the product range covered by this ISDA supplement is not necessarily viewed as equivalent by all market participants. Moreover, the set of first-generation exotics would then change each time ISDA publishes a new supplement. My personal preference is to classify the set of first-generation exotics by the time of introduction in the market.

In my seminars I conclude the section on exotics usually with the Exotics Pedigree in Figure 1.

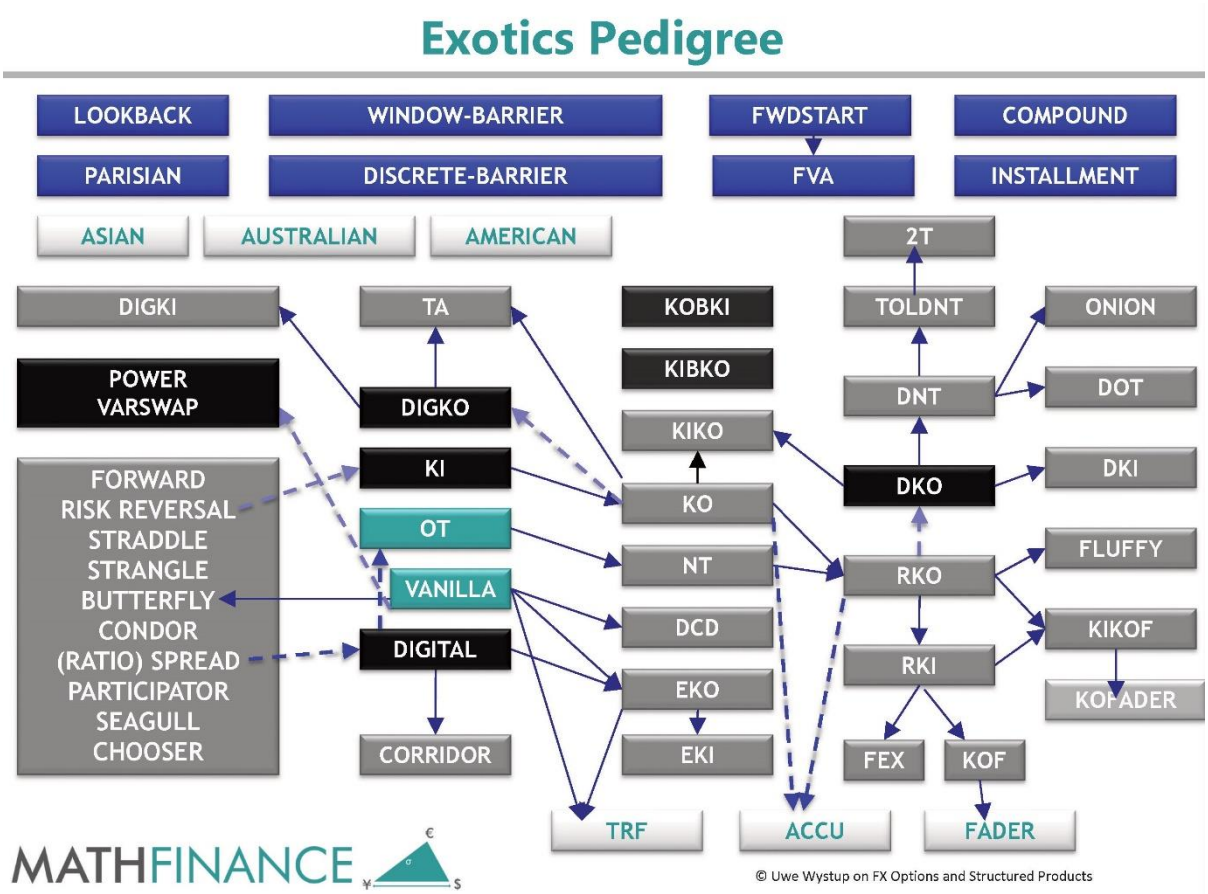


Figure 1: Pedigree of Exotics

[1] ISDA. 2005 Barrier Option Supplement. 2005.
[2] ISDA. 1998 FX and Currency Option Definitions. 1998.

It shows that for most first-generation exotics, the main building blocks are vanilla options and a one-touch contract (OT). **So, in essence, there is only one first generation exotic contract, which is the one-touch. All the rest is built from these and vanilla options**. Let me explain:

A full arrow indicates a perfect replication, for example, a long no-touch contract can be replicated by a long zero-coupon bond and a short one-touch contract, or the barrier knock-in-knock-out symmetry.

$$\text{Knock-In} = \text{Vanilla} - \text{Knock-Out}$$

The entire range of vanilla structures can be replicated by replicated by vanilla options, as shown in Figure 2.

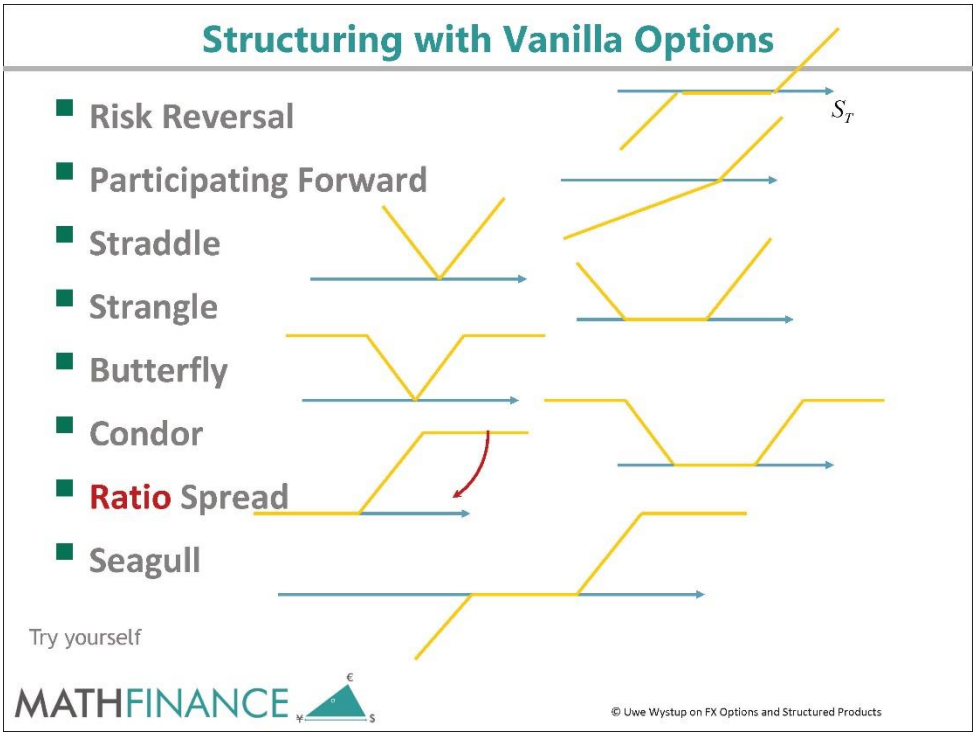


Figure 2: Common Structures using Vanilla Options

Digital Paying Foreign Currency: In FX, the digital call pays a notional amount of a currency if the spot at maturity is at or above the spot (and zero otherwise). This is a path-independent payoff, but the value depends on the currency in which the notional is paid. We know or we can derive by staring at payoff graphs long enough that we have the perfect replication:

European Foreign Digital = European Domestic Digital + Vanilla

Reverse Knock-Out Barrier Options: This is an example of an option that has been discussed intensively in the literature, especially practitioners' literature, for its challenges in hedging exploding Greeks. However, the RKO is nothing new. It can be perfectly replicated with No-Touch (NT) and Regular Knock-Out (KO) barrier options. Recall that in a reverse knock-out (RKO) the barrier is in the money, whereas in a regular knock-out (KO) it is not in the money. The replication of a RKO call with strike K, and barrier B is explained in Figure 3. The indicator variable Φ I like to use takes the value +1 for a call and -1 for a put. The notional of the NT is the maximum intrinsic value of the RKO in domestic currency.

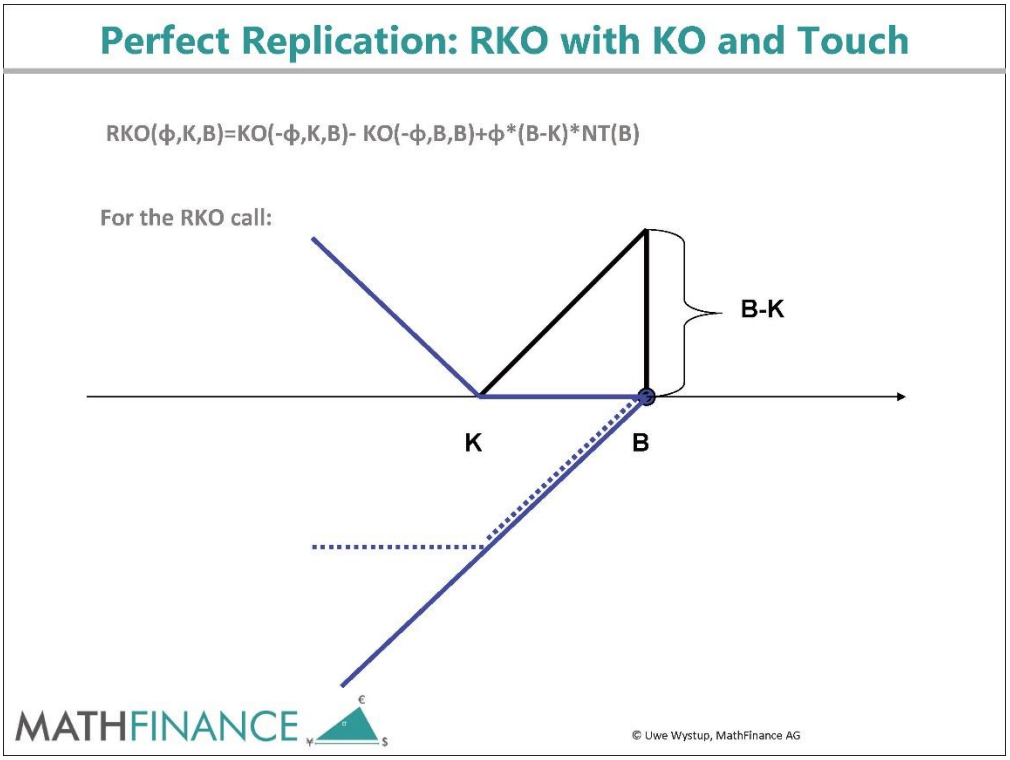


Figure 3: Perfect Replication of a Reverse-Knock-Out (RKO) barrier options with a spread of Regular-Knock-Out (KO) and a No-Touch (NT)

A dotted arrow indicates either an approximation or a semi-static replication. Let me highlight a few examples.

Digital Contracts: As we know a digital call (paying domestic currency) can be replicated by a call spread

$$digital(K) = \lim_{h \downarrow 0} \frac{vanilla(K-h) - vanilla(K+h)}{2h}$$

with strike K and notional 1/(2*h*). Obviously, with *h* being small, the notional blows up, so for all practical matters the limit, while mathematically correct, must be approximated by a ratio with a reasonably small *h*. This makes the replication not perfect, but an approximation.

One-Touch (OT): In FX markets, the one-touch contract pays a notional amount of a currency if the spot trades at or above the continuously observed spot during the lifetime (and zero otherwise). It is like a European digital call, except that it is path dependent. If we denote by S the spot at maturity, and by M the maximum spot observed until maturity, we can write the payoff of a European digital with strike B with indicator functions as $1_{\{S \geq B\}}$ and the payoff of a one-touch with upper barrier B as $1_{\{M \geq B\}}$. These two payoffs are similar, and hence the question comes up how one can replicate a one-touch contract with European digitals and how they are mathematically related.

While I consider the one-touch the key building block of first-generation exotics, it serves itself as an instructive example of a semi-static replication: A trader can sell the one-touch and buy two European digital calls. If spot stays below the barrier B, everything will be out of the money and end with zero cash flows. If and when the spot crosses B, the trader would know at first hitting time τ that he needs to pay 100% of the notional of the one-touch at maturity. He would then sell the two European digital calls and receive two times 50%. The price of the European digital is not necessarily exactly 50% but depends on the forward and the windmill effect from the volatility smile[3]. However, if the spot is at the barrier, the intuitive price of a digital is 50% of the notional.

One-Touch ≈ 2 European Digitals

[3] Uwe Wystup's FX Column "How Can a 50/50 Bet Have Odds of 1:2 Instead of 1:1?", Wilmott, Volume 2018, Issue 98, 14 November 2018, pp. 34-35

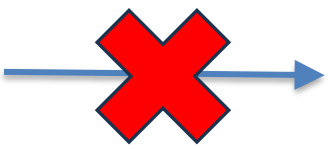
The real price of the European digital at hitting time depends largely on the slope of the smile at that time. This is unknown at time zero and it is not an information that can be backed out from observable vanilla options prices. Thus, while the semi-static replication is highly intuitive, it shows that forward skew is the driving risk, which makes the one-touch a new non-vanilla based instrument, a new Lego block.

Regular Knock-Out (KO) Call: Pricing a regular knock-out call option via a semi-static replication with a risk reversal is one of my favorite exercises in option training. The idea is to replicate a long regular knock-out call option with strike K above the spot (and with the barrier B below the spot) by a long vanilla call with the same strike K and a short vanilla put option with strike F^2/K, where F denotes the outright forward price at hitting time. If the spot does not hit the lower barrier until maturity, the put will be out of the money and the knock-out call will be a vanilla call. If the spot hits the barrier, then the knock-out call is knocked out and worth zero. The risk reversal should be closed at hitting time, so the bid price of the call should be equal to the offer price of the put. At hitting time spot will be on the barrier. We can then apply put-call symmetry to identify the strike price of the put. Call and put option have the same value in the Black-Scholes model if the geometric mean of the strike is equal to the forward. Assuming that the forward at hitting time is near the barrier (because there won't be much time left to maturity), we obtain the rule[4]

$$\text{Regular KO Call} \approx \text{Vanilla Call} - \text{Vanilla Put struck at } B^2/K$$

Furthermore, since at hitting time, we need the value of the risk reversal to be zero, we observe again, that this value largely depends on the prevailing skew of the smile curve at hitting time.

Many more arrows can be found in the Exotics Pedigree in Figure 1. For instance, a DCD (dual currency deposit) is a deposit in combination with a short vanilla options with deferred premium; a KIKO (knock-in-knock-out) barrier options can be replicated perfectly with single- and double-barrier contracts; corridor or range-accrual (RAC) is a strip of digital call spreads; A European knock-out barrier option is a vanilla call spread less a digital contract; A double-no-touch (DNT) can be replicated perfectly by two double-barrier options (DKO); an onion or wedding cake is a set of nested DNTs; a knock-out forward (KOF) is a combination of a long RKO call and a short KO put; even a target forward (TRF) is essentially a strip of KOFs, with a number of adjustments as you go, but structurally, this is what it is.



[4] Details can be found in Foreign Exchange Symmetries, *Contribution to* Encyclopedia of Quantitative Finance, John Wiley & Sons Ltd. Chichester, UK. 2010. pp.752-759.

Out of the Lego Box: Of course, there are exceptions: one group consists of forward-volatility dependent derivatives like compound and installment options, forward-start options, forward volatility agreements (FVA), another group being strongly path dependent derivatives like American, Asian and lookback options. Of course, multi-currency derivatives like baskets or correlation swaps are not built from Lego blocks. However, most liquidly traded FX derivatives trace back to vanilla and one-touch.

Conclusion

1. Vanilla options and on-touch contracts are the key building blocks to create most other exotics.
2. Many exotics can be either approximated or semi-statically replicated by simpler building blocks.
3. When trading exotics, one must get the one-touch price right and/or use observed one-touch prices to calibrate exotics pricing models.