FX Column: OTC Currency Digital Contracts - Traded Price vs. Platform/Model Price

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Recently, Enterprai[1] reached out to us regarding the pricing of the digital contract. When verifying digital prices on the market, the question came up about how to explain a big discrepancy between the prices given by Bloomberg and the prices observed on Depository Trust and Clearing Corporation (DTCC). Before answering this question, let us first consider what a digital call is and how to price it.

European Digital call 101

The digital call is a contract that has a payoff that is characterized as having only two values: either the digital contract is in-the-money (i.e. the spot S_T is at or above the strike K) at maturity T and it will pay a fixed payoff equal to its notional, or the digital contract will expire worthless. The typical payoff is visualised in Figure 1 by the blue line.

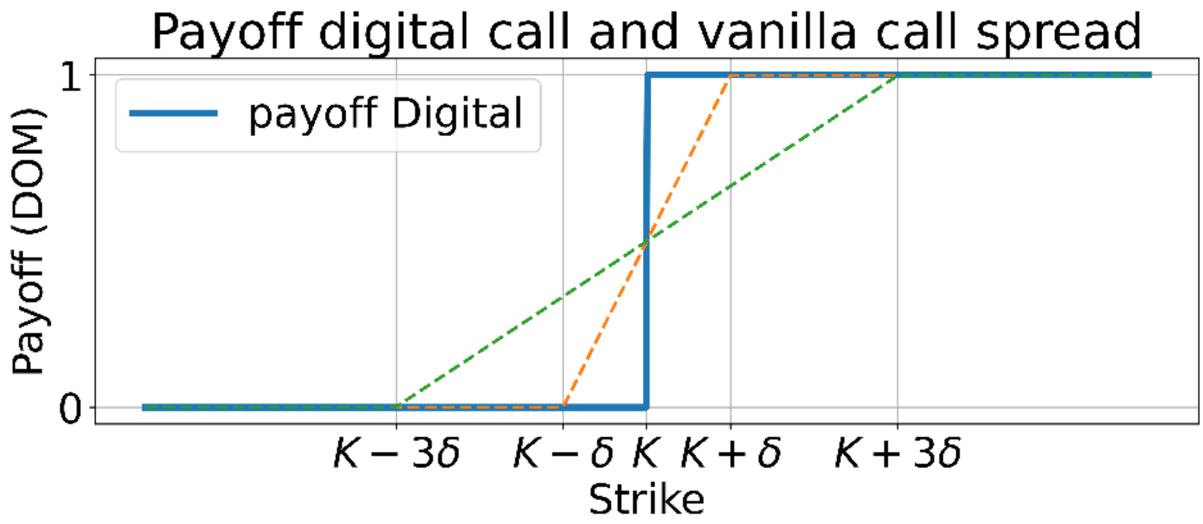

Figure 1: *Approximation of a digital call with vanilla call spreads, 1 unit domestic payoff on the y-axis.*

It is well-known that the payoff of a digital call can be replicated by plain vanilla call options. For the sake of simplicity, let us consider the currency pair USD-JPY with a current spot reference S_0 and some strike K (both in units JPY per USD). If you sell a vanilla call option with strike $K+0.5$ and buy one call with strike $K-0.5$, both with 1 USD notional, the overall payoff at maturity T is

$$\begin{cases} \textbf{0 JPY} & \text{if} \quad S_T < K - 0.5 \\ \textbf{1 JPY} & \text{if} \quad S_T < K + 0.5 \\ \textbf{a value between 0 and 1 JPY} & \text{if} \quad K - 0.5 \leq S_T \leq K + 0.5 \end{cases} \tag{0.1}$$

This payoff is quite similar to that of a digital call with strike K, and, hence, the price of the above described digital call should be near

$$\mathsf{C}(K-0.5,T) - \mathsf{C}(K+0.5,T),$$

where $\mathsf{C}(K,T)$ is the price of a European vanilla call option in JPY per USD.

[1] https://www.enterprai.com/

Instead of buying a vanilla call option with strike $K+0.5$ and selling a vanilla call option with strike $K-0.5$, the more general case is to sell a vanilla call option with strike $K-\delta$ and to buy a vanilla call option with strike $K+\delta$. The result is the following payoff at maturity T:

$$\begin{cases} \textbf{0 JPY} & \text{if} \quad S_T < K-\delta \\ \textbf{2}\delta \textbf{ JPY} & \text{if} \quad S_T < K+\delta \\ \textbf{a value between 0 and 2}\delta \textbf{ JPY} & \text{if} \quad K-\delta \leq S_T \leq K+\delta \end{cases} \tag{0.2}$$

To receive the same payoff as the digital call with a strike K and a payoff of 1 JPY if $S_T > K$, one needs to buy and sell the two plain vanilla call options with a notional of $\frac{1}{2\delta}$ USD. The smaller the choice of δ, the more the payoff will approach that of the digital call, as can be seen in Figure 1 by the dashed green and orange lines. In theory, the price of the digital call, P^{digital}, is thus given by

$$P^{\text{digital}} = \lim_{\delta\to 0} \frac{\mathsf{C}(K-\delta,T) - \mathsf{C}(K+\delta,T)}{2\delta} = -\frac{\partial \mathsf{C}(K,T)}{\partial K}. \tag{0.3}$$

Well, theoretically. In practice we need to choose δ carefully to find the right balance between not having an extremely large notional $\left(\lim\limits_{\delta\to 0^+} \frac{1}{2\delta} \to \infty\right)$ or having a large range of strikes $[K-\delta, K+\delta]$, for which the payoff of the call spread would differ too much from the digital payoff.

The Black-Scholes Model

Now, looking within the Black-Scholes model, let $\mathsf{C}_{\mathsf{BS}}(K,\sigma,T)$ denote the price of a European call option. It is given by

$$\begin{aligned} \mathsf{C}_{\mathsf{BS}}(K,\sigma,T) =& S_0 e^{-r_f T}\mathcal{N}(d_+) - K e^{-r_d T}\mathcal{N}(d_-), \\ d_\pm =& \frac{\ln\left(\frac{S_0}{K}\right) + (r_d - r_f)T \pm \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \end{aligned}$$

where $\mathcal{N}(\cdot)$ is the standard normal cumulative distribution function, σ is the (implied) volatility, r_d the domestic rate and r_f the foreign rate. Equation (0.3) can be rewritten in terms of the Black-Scholes call option prices:

$$\begin{aligned} P^{\text{digital}}_{\mathsf{BS}} &= \lim_{\delta\to 0} \frac{\mathsf{C}_{\mathsf{BS}}(K-\delta,\sigma_1,T) - \mathsf{C}_{\mathsf{BS}}(K+\delta,\sigma_2,T)}{2\delta} && (0.4) \\ &= e^{-r_d T}\mathcal{N}(d_-) \quad \text{JPY}. && (0.5) \end{aligned}$$

Here, $\sigma_- = \sigma(K-\delta)$ is the volatility at strike $K-\delta$, and $\sigma_+ = \sigma(K+\delta)$ is the volatility at strike $K+\delta$. Equation (0.5) contains the standard normal cumulative distribution function, where we can see again the non-discounted price being between 0 and 1 JPY.

Windmill Effect

We have to think carefully about which volatility to use in the above vanilla call option spread. Taking the same volatility for the two vanilla call options within the vanilla call option spread means a replicating portfolio with a flat volatility. Since the smile is not a constant, the result could lead to a significant pricing error, as we would ignore the so-called *windmill effect*, which is visualised in Figure 2. In this specific example, the flat volatility at the level of $K = 115.00$ underestimates the implied market volatility at $K = 114.00$ and it overestimates the volatility at $K = 116.00$.

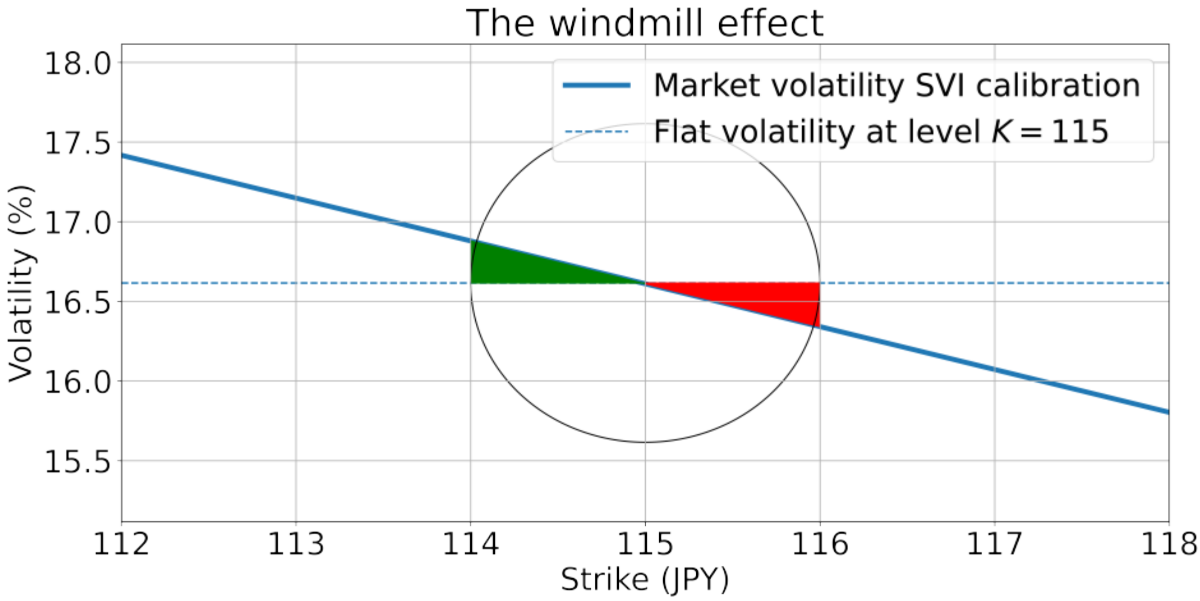

Figure 2: *USD-JPY implied volatility surface on the strike space, zoomed in.*

Digital on Foreign Currency

As if things weren't complicated enough, we could also value a digital call whose payoff is in foreign currency (which is denoted as $\tilde{P}$), or, as in our example of the USD-JPY spot, the payoff is in USD. To retrieve the price for this digital call, one could invert spot and strikes. The digital call could then be approximated by plain European vanilla options with a payoff in USD and a notional in JPY. The formula can then be rewritten in terms of the standard quotation. Setting $K_1 = \frac{1}{\frac{1}{K}+\delta}$, $K_2 = \frac{1}{\frac{1}{K}-\delta}$, the price for the digital call is given by

$$\tilde{P}^{\text{digital}}_{BS} = \frac{\tilde{\mathsf{C}}_{\mathsf{BS}}(K_1, \sigma(K_1, T), T) - \tilde{\mathsf{C}}_{\mathsf{BS}}(K_2, \sigma(K_2, T), T)}{\frac{1}{K_1} - \frac{1}{K_2}} \tag{0.6}$$

where $\tilde{\mathsf{C}}$ is the price in USD per JPY and $\tilde{\mathsf{C}}(K) = \frac{C(K)}{S_0 K}$.
So far the mathematics. A trader thinks of a USD-paying digital call as simply the sum of a JPY-paying digital call and a vanilla USD call JPY put. This is based on payoffs and is therefore model-independent. The important question to ask is "which currency". Traders in equity options distinguish so-called *cash-or-nothing* digitals (the JPY-paying digital in FX) from *asset-or-nothing* digitals (the USD-paying digital in FX).
On a final note to make: pricing the digital put is analogous the above procedure, but with plain European vanilla put options in the replication. More details can be found in the FX column [4] and in [2].

Case Study: USD-JPY Digital Put

Finally, let's take a closer look at a real-world example. We would like to determine the price of a European USD-JPY digital put with strike $K = 115.00$ and a payoff of 5 million USD, with the premium given in USD. The horizon date is March 29, 2023. The spot reference is $S_0 = 132.87$ JPY per USD, time to maturity is 84 days (June 21, 2023) and the delivery date is June 23, 2023. Enterprai spotted a traded price of USD 202,500 in DTCC. A snippet of the market on the horizon date is shown in Table 1.

T	ATM	25RR	10RR	25BF	10BF	Fwd	Rate(USD)
1W	11.340	-1.0825	-1.855	0.2675	0.7050	132.848	5.000
2W	12.255	-1.2000	-2.210	0.2850	0.7755	132.494	5.070
1M	11.695	-1.3350	-2.580	0.3000	0.8400	132.317	5.095
2M	12.485	-1.7700	-3.385	0.3650	1.0200	131.467	5.175
3M	12.500	-1.8100	-3.580	0.3900	1.1450	130.708	5.225
6M	11.735	-1.1532	-3.070	0.4125	1.3150	128.455	5.245

Table 1: *Snippet of the market data on March 29, 2023. T is the tenor, ATM is the at the money volatility, 25RR and 10RR are risk reversal volatility at the 25 and 10 delta respectively; BF is the butterfly (in %). Fwd is the forward rate. Rate is the USD money market interest rate for deposits in %.*

Taking the market mid-price of USD 168,300 in Bloomberg as a reference, one might be surprised that the traded price is more than 20% above the mid-price. However, this can be explained by the fact that the digital is far out of the money. Generally, interbank bid-offer spreads for European digitals in USD-JPY are about 2-3% of the notional. The notional being 5 million USD, the mid-offer difference would be at least 1%, so 50,000 USD. Therefore, the offer price based on Bloomberg is expected to be around 218,300 USD (typically with an additional sales margin). Compared to that, the traded price is actually lower than expected. Many FX options traded OTC are reported in DTCC, particularly those traded in US-based banks with their clients. On the Enterprai platform, users can see an aggregate market monitor, to get an overview of the vanilla or exotics market and the respective delta and gamma exposure, see Figure 3.

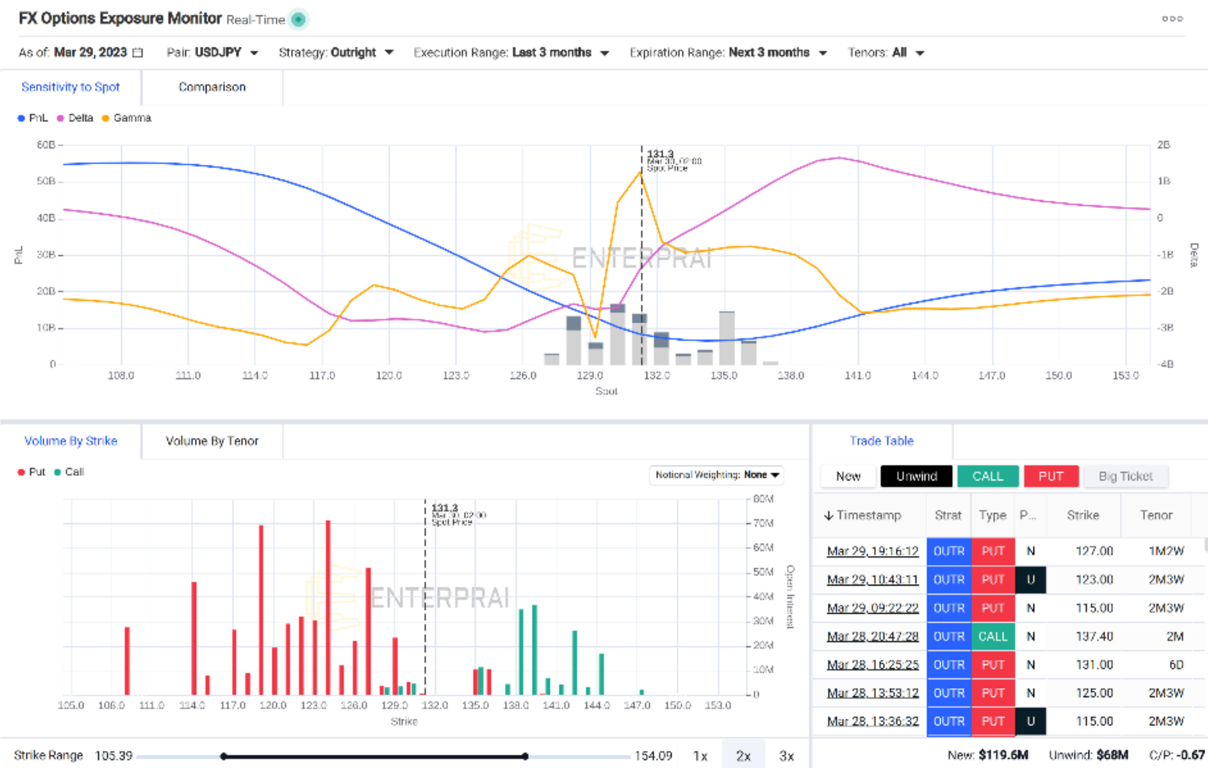

Figure 3: *Exposure Monitor of the Enterprai platform. Users can retrieve traded prices of OTC FX Options. The methodology goes as following - we select only trades executed within last three months, as of March 29 and expiring within the next 3 months. We also look only at stand-alone contracts (called* outrights *on Enterprai), i.e., spreads, calendar spreads, risk reversals and straddles are excluded from the analysis.*

In Figure 4, we show the implied volatility curve visualised for the expiry date June 21, 2023, using stochastic volatility inspired (*SVI*) parameterization.

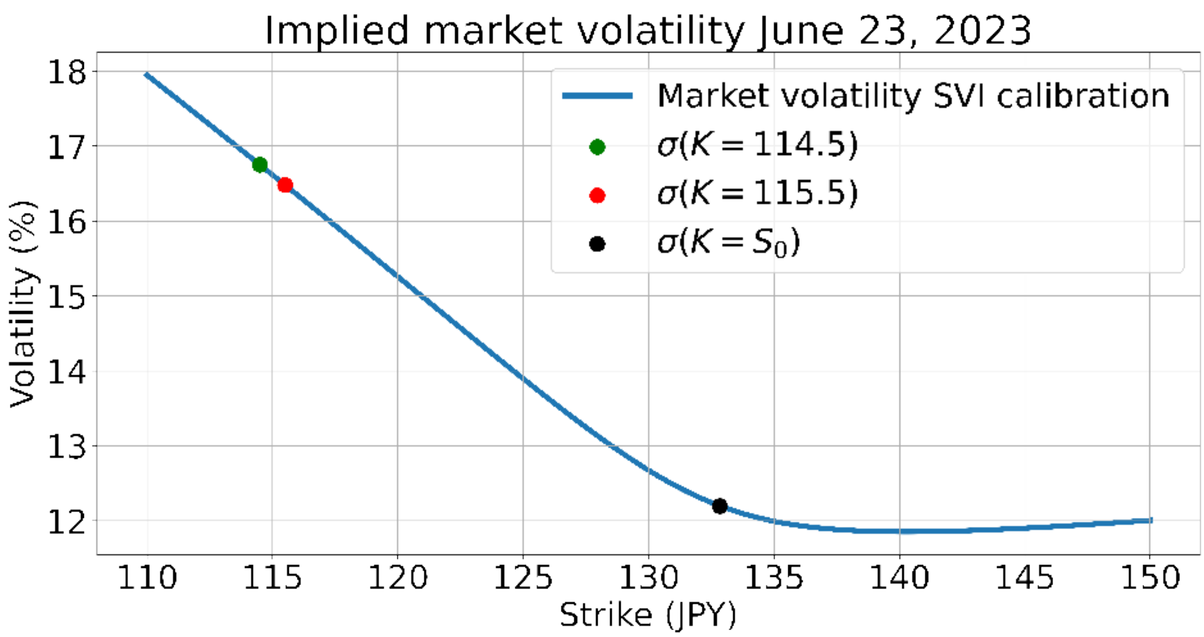

Figure 4: *USD-JPY implied volatility surface on the strike space.*

	Bloomberg	%USD	MathFinance	%USD	Eikon	%USD
Digital $\boldsymbol{BS}$	\$168,300	3.37%	\$152,000	3.04%	\$218,600	4.07%
$P(\delta = 0.5)$	\$168,600	3.37%	\$152.100	3.04%	\$158,700	3.17%
$P(\delta = 0.05)$	\$168,000	3.36%	\$152,000	3.04%	\$158,700	3.17%
Digital $\boldsymbol{SLV}$	\$160,000	3.20%	\$151,900	3.04%		
$P(\delta = 0.5)$	\$162,000	3.24%	\$152,100	3.04%		
$P(\delta = 0.05)$	\$162,600	3.25%	\$150,800	3.02%		
Digital $\boldsymbol{VV}$	\$188,000	3.20%	\$182,500	3.65%	\$179,900	3.60%
$P(\delta = 0.5)$	\$188,100	3.24%	\$182,600	3.65%		

Table 2: *Prices for a digital put with strike $K = 115.0$ and spot date March 29, 2023. The notional is 5M USD, the payoff is also in USD. $P(\delta = 0.5)$ and $P(\delta = 0.05)$ are the put spreads with strikes $K = [114.5, 115.5]$ and $K = [114.95, 115.05]$, respectively, priced under different models written in bold: **BS** is the Black-Scholes model, **SLV** stochastic local volatility model and **VV** the Vanna Volga model.*

The resulting digital call prices are summarised in Table 2. Here we price the digital put directly with the above-mentioned plain vanilla put option spreads. We would expect that on a commercial pricing platform, the put spread prices approach the price of the digital put. Eikon does not seem to follow this rule.

What is interesting to note in the Bloomberg results is that the price of the digital put in the smile model does not coincide with the price in their SLV (stochastic-local volatility) model. In an SLV model, all vanilla options are priced to the market. We suspect that the difference between USD 160,000 (SLV) and USD 168,300 (Black-Scholes with Smile) is because of the extrapolation below the 10% delta level, and Bloomberg might be using different extrapolation schemes for the smile in their Black-Scholes model and their SLV model. Eikon, on the other hand, does not seem to price the digital put correctly in the Black-Scholes model, but its vanna-volga-price seems more reasonable. More on the FX exotics models can be found in [3].

The Slope of the Smile on the Strike-Space Matters

First we realize that the digital put struck at $K = 115.00$ is far out-of-the-money. This strike is in fact at a vanilla-delta level near -3%. The typical market input for vanilla options is down to the -10% delta. Therefore, the volatility for strike $K = 115.00$ is on the part of the volatility smile curve that depends on the choice of extrapolation.

As we have seen from the pricing via the representation by vanilla spreads, not only the *level* of the implied volatility smile at the strike level, but also the *slope* of the inter- or extrapolation curve at strike K enters the value of a digital. Ignoring the effect of the slope means to keep the volatility within the vanilla put spread constant (for example, the volatility at the center strike $K = 115.00$).

The digital put price observed in Eikon does not seem to include the windmill-adjustment. This adjustment is negative, as the slope of the smile curve on the strike-space is negative and we price a digital put. The approximation with a vanilla put spread clearly shows this.

USD-ZAR One-Touch Barrier Chasing

Digital contracts can have extreme deltas, and so do their path-dependent versions, the one-touch and no-touch contracts. On Enterprai one might spot some digital contracts, which will then explain high activities in the spot trading, right before maturity and when the FX spot is close to the barrier. Some market participants go even further and try to trade spot to push the spot towards the barrier. An example of this has been reported recently in Joe Parsons in Risk.net [1], where in fact, a criminal case was brought against a hedge fund for allegedly manipulating the US dollar/South African rand spot rate to trigger a one-touch. Maybe more on this in the next FX column.

Conclusions

1. Pricing platforms sometimes show different prices for exotics, even simple exotics like digitals. Prices are essentially driven by the interpolation and extrapolation of the smile.

2. Before trading on a price, it is advisable to double check the price of a digital via a call- or put spread approximation, as this will include the windmill-effect.

3. Traded prices of OTC FX options can be filtered and illustrated on the Enterprai platform. The prices there include bid-offer spread and sales margin. Digital contracts' prices can trade at much higher amounts than the mid-market value, especially when the contracts are far out-of-the money.

Bibliography

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