

## Derivatives Educational Column: Shorts Should not be Worn too Long

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**Surprise Surprise:** A DAX-short investor **gains 1.2%** while the DAX **index falls by 2.8%** and is shocked. Because: The idea is that if the index falls by 10%, the short rises by 10%. This column is about explaining what's going on. Investing in falling underlyings, e.g., spot indices, can be achieved by buying so-called "Shorts". Of course, institutional investors can short sell the underlyings directly, but this requires a professional setup with a credit line. Selling index futures would be another alternative, but one that requires a margin account. Shorts as investment certificates are widely used in the retail market. However, one needs to understand the details.

**Short DAX Formula:** As a stock index I use DAX, the German major stock index. This is just an example, and the idea translates to any underlying. Replicating the opposite percentage change is typically done via the formula in [Figure 1](#).

$$Index_t = Index_T \left[ 1 - \left( \frac{DAX_t}{DAX_T} - 1 \right) \right] + Index_T (2EONIA_T - C_M) \frac{d}{360}$$

**Figure 1: Index Formula for a Short**

In the example of the Comstage ETF004, which started on 29 December 2006 at a level of 6,596.92, the building blocks mean:

- ▲  $Index_t$ : the short DAX index at time t,
- ▲  $Index_T$ : DAX closing price on the previous day,
- ▲  $DAX_t$ : DAX level at time t,
- ▲  $DAX_T$ : DAX closing level on the previous day,
- ▲  $EONIA_T$ : EONIA rate on the previous day,
- ▲  $C_M$ : monthly borrowing cost of the underlying index, called the repo rate
- ▲ d: the number of days between t and T.

The first term is called the short part, the second the compounding part. The first short part can be interpreted as

If DAX is up by p% from the previous day, then the ShortDAX falls by p% (and vice versa).

Example: Let  $DAX_t=10,000$ ,  $DAX_T=9,900$ ,  $Index_T=25$ . Then the percentage change is  $DAX_t/DAX_T=0.01=1\%$ . Taking off 1% of the ShortDAX index yields  $Index_t=24.75$ . So far, so good.

**Investment vehicle:** ShortDAX is an index published by the German Exchange<sup>1</sup>. However, investment in this index requires a vehicle, such as an ETF, e.g., ETF004 issued by Société Générale (formerly Commerzbank) or DBX1DS issued by Deutsche Bank. As usual ETFs come with a tracking error compared to the reference index, which the issuers try to keep small.

**Illustrative Example:** Suppose DAX is at 8,800 points, and an investor buys shorts for EUR 8800. A drop of 1% in DAX means 88 points down to 8,712. The Short rises by 1%, so 88 points up to 8,888. So, if DAX is down by 88 points, ShortDAX is up by 88 EUR. However, if DAX is at 16,100 points and the ShortDAX ETF has a value of 4,400 EUR, and DAX drops by 1%, i.e., by 161 points, the ShortDAX rises by 1%, which is 44 EUR. As a result, DAX loses 161 points, but ShortDAX gains “only” 44 EUR.

Shorts don't mimic absolute differences, but opposite percentage differences.

I illustrate in [Figure 2](#) historic data for the DBX1DS Short vs DAX index. It clearly shows that levels of DAX can have different levels of ShortDAX. Moreover, the colors seem to indicate that for the same DAX level, the price of the ShortDAX tends to go down.

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<sup>1</sup> <https://www.boerse-frankfurt.de/index/shortdax/kurshistorie/historische-kurse-und-umsaetze>

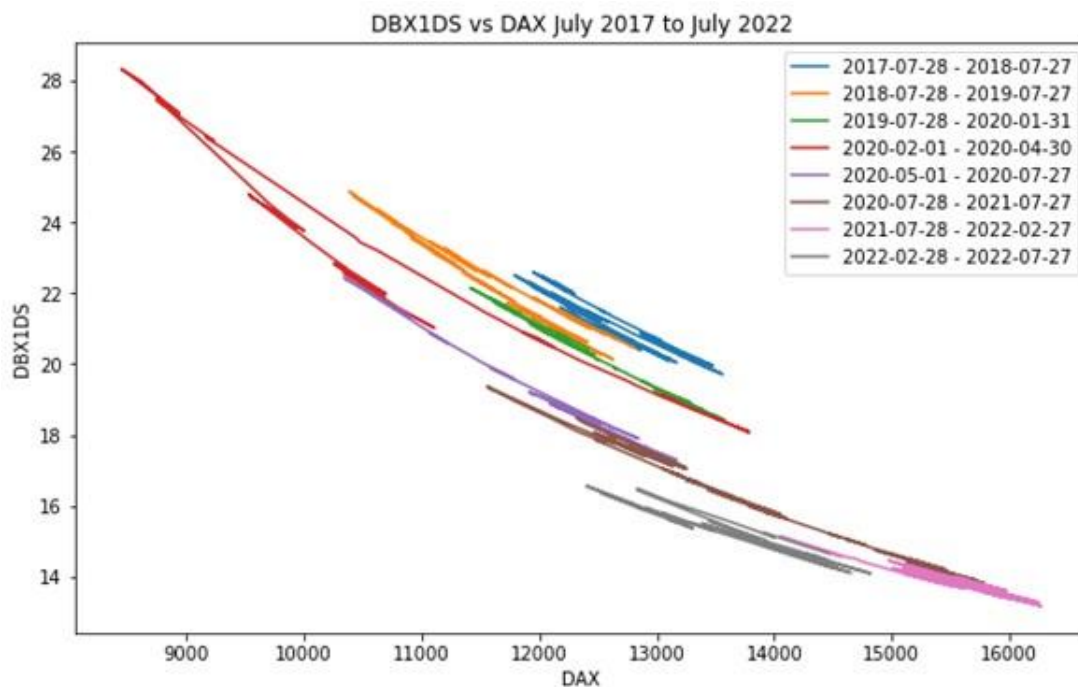


Figure 2: DAX<sup>2</sup> index vs. ShortDAX ETF DBX1DS<sup>3</sup>

We observe that there is a reciprocal relationship between the DAX and the Short for certain time intervals, but for longer holding periods the reciprocal relation tends to differ. Warning: As a Short investor, when holding the Short for a long time, you will never make as much profit as the DAX loses. You will more likely suffer from Murphy's law and always lose more than expected.

Shorts shouldn't be held to long.

**Alternative Index:** For a standard Short we have the rule that if the initial value 100 first rises by 5% and then drops by 5%, we land at  $100 \times 1.05 \times 0.95 = 99.75 < 100$ . Thus, a permanent up and down causes losses, even if up and down percentages are symmetric. An alternative index that takes you back to 100 would be up by 5% and down by  $1/1.05$ , in which case we would by construction obtain  $100 \times 1.05 \div 1.05 = 100$ . However, this is not the way Shorts are designed. This simplified rule is illustrated in Figure 3.

<sup>2</sup> <https://www.boerse-frankfurt.de/index/dax/kurshistorie/historische-kurse-und-umsaetze>

<sup>3</sup> <https://de.finance.yahoo.com/quote/DXSN.DE/>

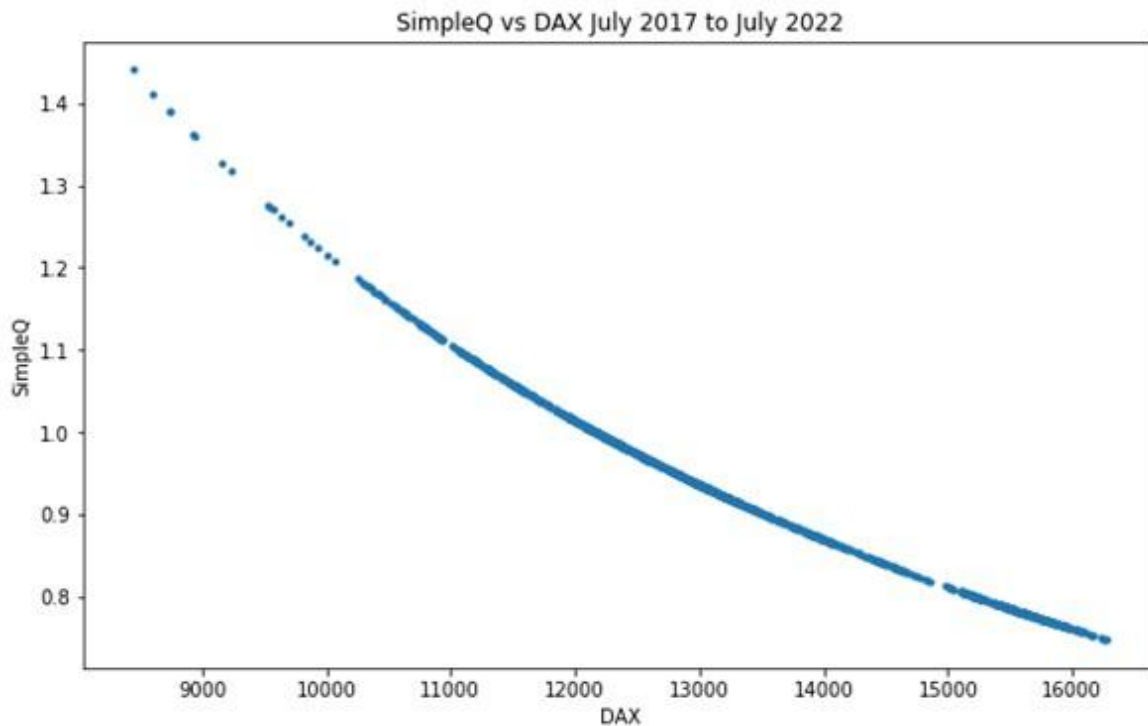


Figure 3: DAX index vs. alternative simplified ShortDAX.

**The Underlying Math: Step by Step Examples:** We start with an initial value of  $d_0 = 1$  for the DAX index and  $s_0 = 1$  for the corresponding Short.

- (a) 1-day change by 2%. Index value after one day is 1.02 for DAX and 0.98 for Short. The percentage change is +2% vs. -2%. In formulas we get  $d_1 = 1 \times (1+0.02)$ ,  $s_1 = 1 \times (1-0.02)$ .
- (b) 2-day change by 2% up and down. Both indices land at the same value:

$$d_2 = d_1 \times (1-0.02) = 1 \times (1+0.02) \times (1-0.02) = 1.02 \times 0.98 = 0.9996$$

$$s_2 = s_1 \times (1+0.02) = 1 \times (1-0.02) \times (1+0.02) = 0.98 \times 1.02 = 0.9996$$

Phenomenally, DAX is down by 0.04% and ShortDAX is also down by 0.04%.

A loss in DAX does not imply a gain in the Short.  
Both can lose value.

The key insight is that the percentage change on the second day does not take the initial value as a reference. Denoting by  $x_i$  the percentage change of day  $i$ , we get for two days:

$$d_2 = (1+x_1) \times (1+x_2) = 1 + x_1 + x_2 + x_1 \times x_2 \neq 1 + x_1 + x_2$$

In the example  $x_1 = 2\%$  and  $x_2 = -2\%$ , so

$$d_2 = (1+0.02) \times (1-0.02) = 1 + 0.02 - 0.02 + (0.02) \times (-0.02) \neq 1 + 0.02 - 0.02.$$

The final percentage change of ShortDAX is not equal to the sum of the negative percentage changes of the DAX index.

- (c) 2-day change by 10% up and 8% down. DAX lands at 0.972, ShortDAX at 1.012. The percentage changes are -2.8 for DAX and +1.2 for ShortDAX, not absolute images of each other, the difference being 1.6% lower than a naïve estimate of -2.8% also for the ShortDAX. The cross term  $x_1 \times x_2$  impacts the difference and can yield a substantial difference if daily percentage changes are. The size of the impact is a consequence of the design of the cross term, or cross-terms in general, when we have more than two days. Let's consider this next.

**The Underlying Math in General:** Let's consider 5 days (5 is general) with daily percentage changes  $x_1, \dots, x_5$ . Then we get for DAX and ShortDAX

$$d_5 = (1+x_1) \times (1+x_2) \times (1+x_3) \times (1+x_4) \times (1+x_5)$$

$$= x_1x_2x_3x_4x_5 + x_1x_2x_3x_4 + x_1x_2x_3x_5 + x_1x_2x_3 + x_1x_2x_4x_5 + x_1x_2x_4 + x_1x_2x_5 + x_1x_2 + x_1x_3x_4x_5 + x_1x_3x_4 + x_1x_3x_5 + x_1x_3 + x_1x_4x_5 + x_1x_4 + x_1x_5 + x_1 + x_2x_3x_4x_5 + x_2x_3x_4 + x_2x_3x_5 + x_2x_3 + x_2x_4x_5 + x_2x_4 + x_2x_5 + x_2 + x_3x_4x_5 + x_3x_4 + x_3x_5 + x_3 + x_4x_5 + x_4 + x_5 + 1,$$

$$s_5 = (1-x_1) \times (1-x_2) \times (1-x_3) \times (1-x_4) \times (1-x_5)$$

$$= -x_1x_2x_3x_4x_5 + x_1x_2x_3x_4 + x_1x_2x_3x_5 - x_1x_2x_3 + x_1x_2x_4x_5 - x_1x_2x_4 - x_1x_2x_5 + x_1x_2 + x_1x_3x_4x_5 - x_1x_3x_4 - x_1x_3x_5 + x_1x_3 - x_1x_4x_5 + x_1x_4 + x_1x_5 - x_1 + x_2x_3x_4x_5 - x_2x_3x_4 - x_2x_3x_5 + x_2x_3 - x_2x_4x_5 + x_2x_4 + x_2x_5 - x_2 - x_3x_4x_5 + x_3x_4 + x_3x_5 - x_3 + x_4x_5 - x_4 - x_5 + 1.$$

Consequently, the percentage total difference in change after 5 days is (the some of the two terms above:

$$2x_1x_2x_3x_4 + 2x_1x_2x_3x_5 + 2x_1x_2x_4x_5 + 2x_1x_2 + 2x_1x_3x_4x_5 + 2x_1x_3 + 2x_1x_4 + 2x_1x_5 + 2x_2x_3x_4x_5 + 2x_2x_3 + 2x_2x_4 + 2x_2x_5 + 2x_3x_4 + 2x_3x_5 + 2x_4x_5$$

Besides the 5 summands with 4 factors, which are expected to be comparatively small, there are 10 summands with two factors; those drive the show. In general, after  $n$  days, there will be  $n \times (n-1) / 2$  summands with 2 factors. Notice that the impact is not path-dependent but depends on how many of these summands cancel each other because their sign may be different. For many observation days, these factors might add up to a substantial amount.

For example, if we have 9 days, 6 down by 1%, 3 up by 1%, DAX lands at 0.97, short at 1.03. The total percentage changes are -3% for DAX and +3 for ShortDAX. If we use 10% instead of 1%, we arrive at a total percentage change of -29.27% for DAX and +29.15% for ShortDAX, still rather close in terms of size. So, it isn't the size of the daily changes, nor the sequence, but the details of the factors and their signs.

For example, if we have 12 days, 6 down by 10%, 6 up by 10%, the total percentage change is +3.56% in DAX and -15.27% in ShortDAX. It's these scenarios that should alarm investors when they think of how to benefit from a falling stock index. One can prevent these surprises by resetting the notional back to 1 after each day, i.e., one would need to invest the same amount of cash every day. This may not be practically viable, as the bid-offer spread might ruin the idea.

The Difference in an investment in the DAX index and going short the ShortDAX during the COVID pandemic is illustrated in [Figure 4](#).

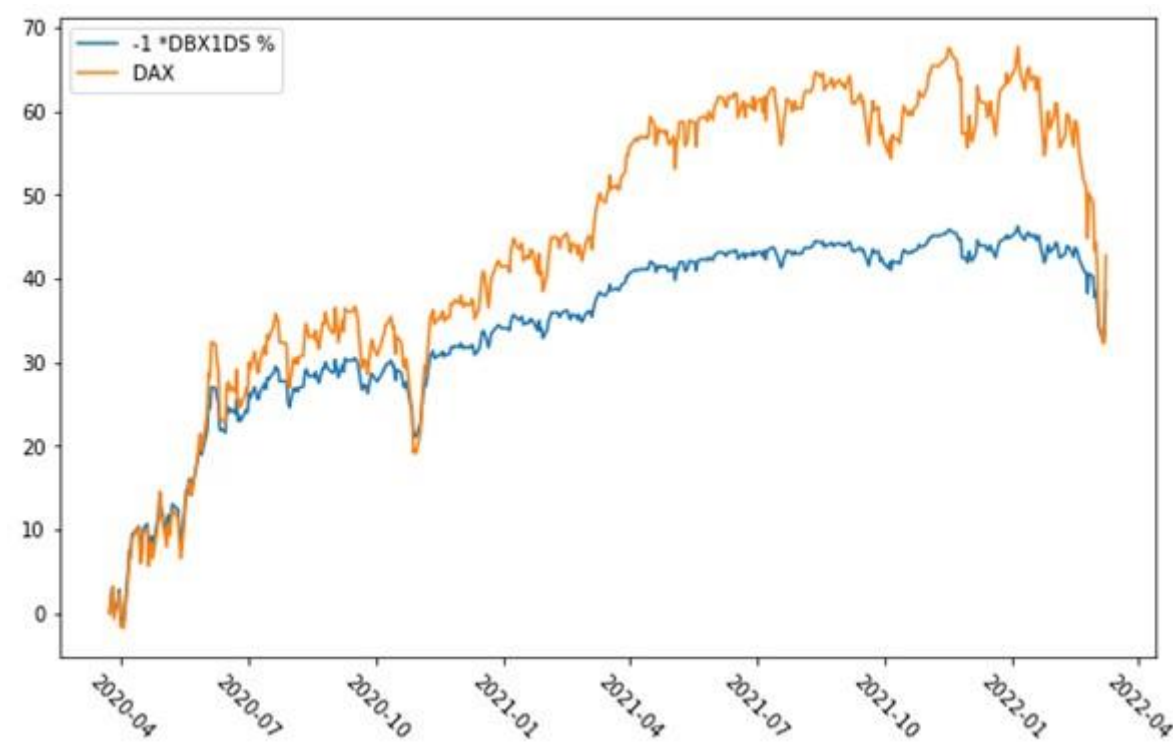


Figure 4: Percentage total growth compared: DAX vs. Short the ShortDAX DBX1DS

## Conclusion

1. Equity index short ETFs can silently lower your naïve expected performance when held for a long time.
2. Do the math before you invest.
3. The distribution of cross-terms drives the performance, and there is no intuitive way to capture the effect.
4. Rebalance your investment to constant amount of cash invested can prevent this effect.