# FX Column: How a Long Call Option can be Long Gamma, Long Theta and Short Theta

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In my options training courses, I often ask participants if a long vanilla option can be *both long gamma and long theta*. Normally, options textbooks train us that the value of an option is the sum of its intrinsic value and the time value. As time progresses, the value converges to the payoff function, and the time value converges to zero. It is one of the common misconceptions[1] that time value is always positive. The forward curve goes down (backwardation), the time value can be negative for in-the-money options, i.e., the value of the call option is less than the payoff function. And in FX markets, this is quite common. As an example, we consider a market in USD-JPY, where USD interest rates are higher than JPY interest rates, and therefore, the forward curve decreases. With a negative time-value, we would then conclude that a deep in-the-money long USD call JPY put option is long gamma (because the payoff is convex) and long theta, because the value is below the payoff and converges up to the payoff.

Today, we stretch this phenomenon a bit further and show that such a USD call can be short theta after all and will further learn that it can be *both short theta and long theta*. No kidding.

**The market** I consider is USD-JPY spot 127.00 on the trade day May 26, 2022, with spot date May 3USD 6-months money market interest rate 1.77%, JPY 6-months money market interest rate -0.40% (yielding a 6-months forward rate of 125.61 below initial spot), ATM volatility 9.634%, 25-delta risk reversal -0.654% (favoring USD puts), 25-delta butterfly 0.441% (standard). This is a real example.

**The option** I consider is a 6-months USD call JPY put struck at 107 with 1M USD notional, which has an expiry date on November 28, 2022, and a delivery date on November 30, 2022. Quick recap: May 26, 2022, is a Thursday; adding two business days will take us to Monday, May 30, 2022. This is the last Monday in May, and hence Memorial Day in the US, a public holiday; consequently, the spot date is Tuesday, May 31, 2022. That being the last day of the month will make the 6-months delivery date the last day of November, so Wednesday, November 30, 2022. The expiry date is two business days before on Monday, November 28, 2022. It is because of this Memorial Day, that the 6-months option on the next trade day, Friday, May 27, 2022, will have the same expiry and delivery dates. Amazing. I really hope that the people in the US enjoy their holiday to the fullest. Few will know the impact on USD-JPY options. Figure 1 shows the price of the option in SuperDerivatives on the two trading days.

**The observations** are: (1) The intrinsic value of the option with strike 107 and spot reference 127 is 20 JPY per USD, so JPY 20M. (2) The value shown in SuperDerivatives is JPY 18,890,895, which is lower than the intrinsic value as expected, and therefore one would expect a positive theta. (3) However, SuperDerivatives shows a negative JPY 1,894 JPY theta. At first, I thought this was a bug. (4) However, if we move the trade date in SuperDerivatives to the next day, keeping all other market and contract values the same, the option value does in fact decrease to JPY 18,889,350. Hence, the theta shown in SuperDerivatives is consistent with the option prices[2]. I note that this is the notion of theta a trader usually thinks of: what happens to the position on the next day, assuming market variables don't change. As a quant one might rather think of theta being the derivative of

[1] Wystup, U. FX Column „Shedding Light on Common Misconceptions“, Wilmott, issue 104, Nov 2019, pp. 16-19.

[2] Special thanks to Benjamin Dana and Jonathan Binke at ICE Data Services, who helped me clarify what's happening in SuperDerivatives.

the option value with respect to running time. This will yield a change per year. To get to the traders' theta, we would need to divide the quants' theta by 365. In practice, however, there are all the matters around trade days, business day, holidays, and currency pair specific conventions. One can't just calculate an annual change, divide by 365 and expect to get the same answer.

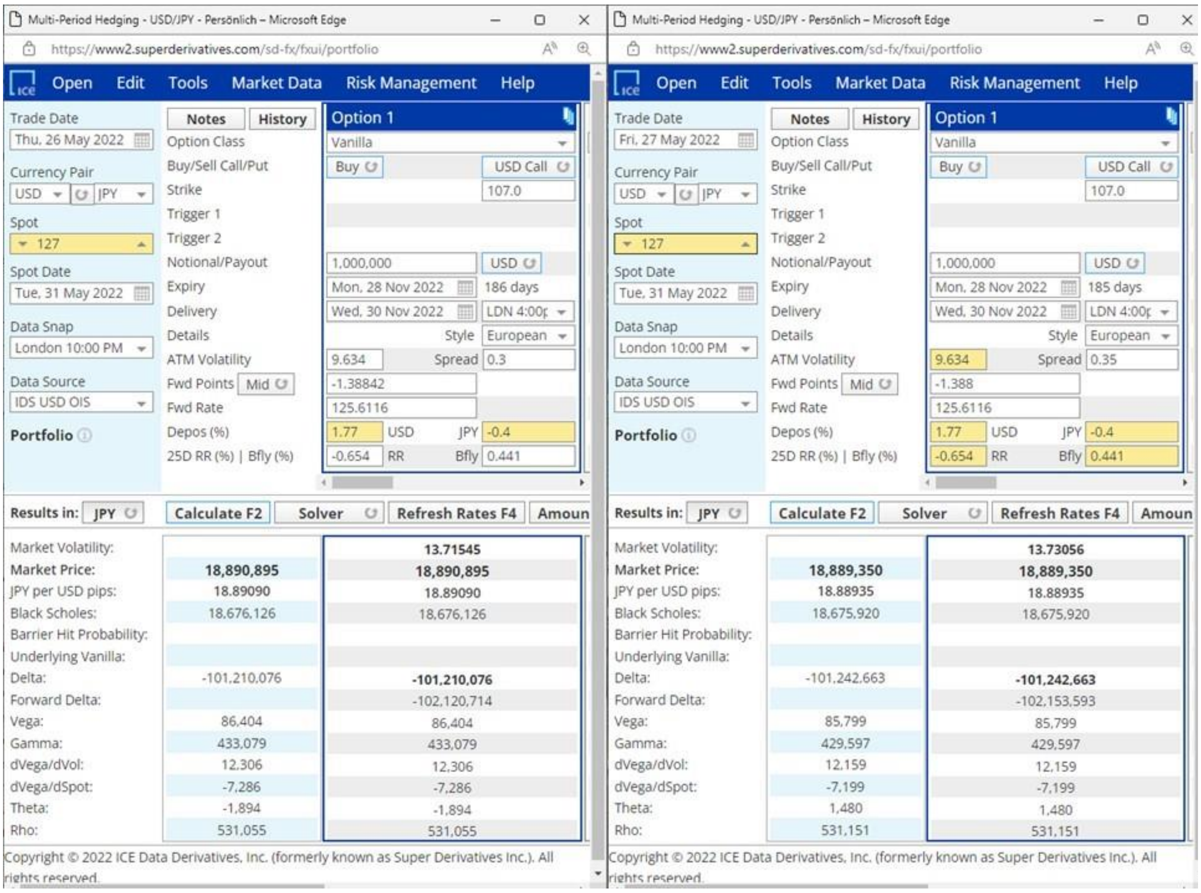


**Figure 1: Pricing the USD call in SuperDerivatives on May 26, 2022, and the next day May 27, 2022.**

(5) A further observation is that on May 27, 2022, the same option does in fact have a positive theta, so we are back on familiar grounds.

**Comparison with Eikon's FX Option Calculator**

If we price the same USD call option with identical parameters in Eikon, we get the results shown in Figure 2. In particular, theta is 4,064 JPY, so the USD call option is long theta as one would naively expect it. This theta does not seem to consider the business day calendar but is more likely derived from a mathematical derivative. It is also informative to consider such Black-Scholes Greeks with some input for volatility and rates and daycount fractions. This would then show the user intuitively an average daily change of the option value ignoring the up- and down jumps of theta when business days are considered in detail. Both approaches might be useful to look at. The take-away message is that a clear definition of theta should be stated along with the size of theta, so the user knows what exactly is shown.

**The Math Behind**

The Black-Scholes formula to compute the value of a call option is shown in Figure 3. It generalizes the academic version of the formula to two different time spans: t is the time for the volatility uncertainty, i.e., it starts at the trade date and ends at the expiry date (and time). T is the time for discounting between cash flows, i.e., it starts at the spot date and ends at the delivery date.

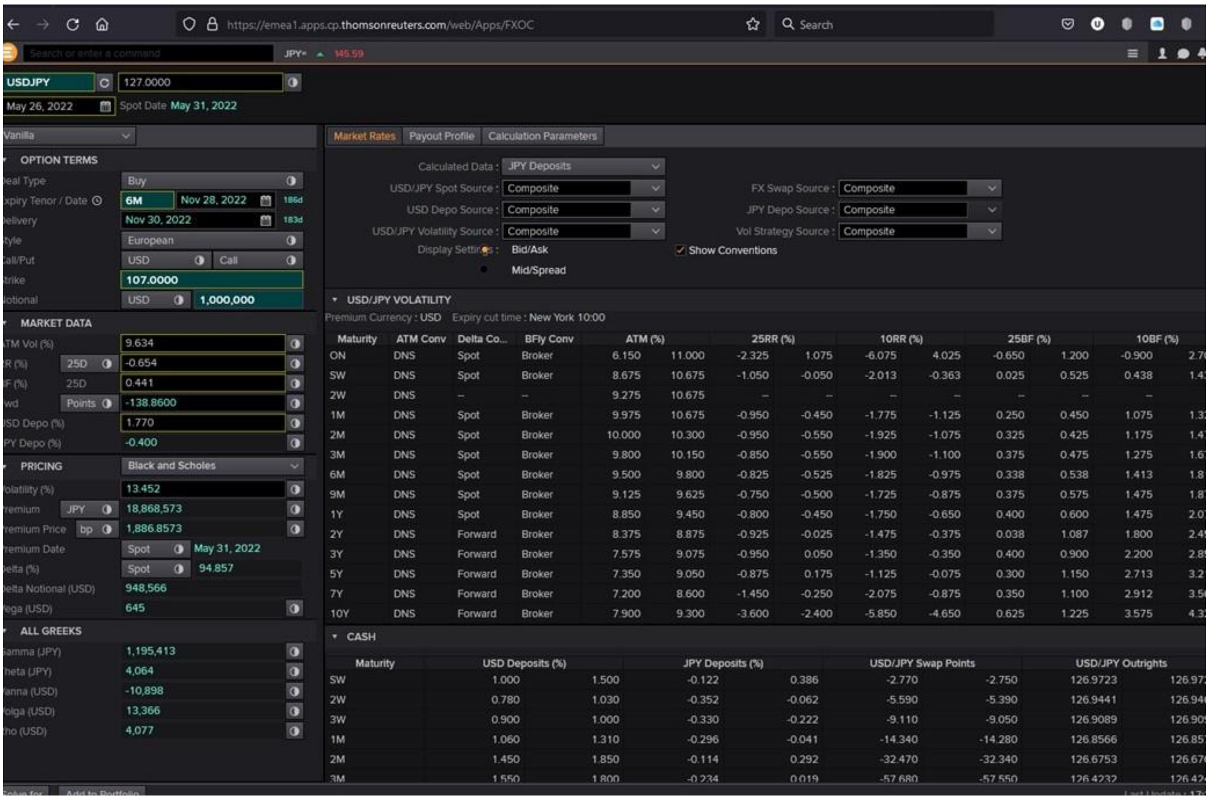


Figure 2: Pricing the USD call in Eikon on May 26, 2022.

$$v(0) = e^{-r_d T}\left[f_0(T)N(d_+) - KN(d_-)\right]$$
$$= S_0 e^{-r_f T} N(d_+) - K e^{-r_d T} N(d_-)$$
$$d_\pm = \frac{\ln \frac{f_0(T)}{K} \pm \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$
$$= \frac{\ln \frac{S_0}{K} + (r_d - r_f)T \pm \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

**Figure 3: Black-Scholes Formula: Value of a Call Option Using a Volatility-t and a Rates-T[3]**

The other symbols in the formula are the usual K for strike, S for spot, σ for volatility, $r_d$ for domestic (JPY) rate, $r_f$ for foreign (USD) rate, f for forward rate, v(0) for value at inception, N for the cumulative normal distribution function. As a quant student starting with options, there is usually no difference made between the vol-t and the rates-T; and 6-months is just simply 0.5 years. In our above example of the USD call, t becomes one day smaller when we move to the next trade day, but T does not change, as spot and delivery date do not change. The theta shown in SuperDerivatives only shows the impact of the "optionality", but not the impact caused by the forward curve, which is actually the dominating one on average. Try it yourself and extend your personal options calculator to handle different t and T time intervals.

**Conclusion**

1. Any theta figure should come with a clear definition what it means.
2. You can consult different pricing and risk management systems for the same market and same option and get different answers even for standard options, in particular, there can be a vanilla call option long gamma, long theta and short theta.
3. You can learn more about theta on the next MathFinance Conference 13-14 March 2023 in Frankfurt. This is relevant for P&L-Explain.



[3] Wystup, U. FX Options and Structured Products, 2nd Edition, Wiley 2017, p. 29.