Volatility as investment - crash protection with calendar spreads of variance swaps

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Abstract. Nowadays, volatility is not only a risk measure but can be also considered an individual asset class. Variance swaps, one of the main investment vehicles, can obtain pure exposure on realized volatility. In normal market phases, implied volatility is often higher than the realized volatility will turn out to be. We present a volatility investment strategy that can benefit from both negative risk premium and correlation of variance swaps to the underlying stock index. The empirical evidence demonstrates a significant diversification effect during the financial crisis by adding this strategy to an existing portfolio consisting of 70% stocks and 30% bonds. The back-testing analysis includes the last ten years of history of the S&P500 and the EUROSTOXX50.

Introduction

Volatility is an important variable of the financial market. The accurate measurement of volatility is important not only for investment but also as an integral part of risk management. Generally, there are two methods used to assess volatility: The first one entails the application of time series methods in order to make statistical conclusions based on historical data. Examples of this method are ARCH/GARCH models. The second method is based on implied volatility from option prices, which reflects the investor's view on future volatility. Historically, volatility used to be traded through options or options strategies such as butterflies or strangles. Using such a strategy exposes investors to the risks of changes in implied volatility. However, holding option strategies also exposes investors to risks arising from changes in other underlying markets, for example, interest rates and the underlying (referred to by the Greek letters Delta, Gamma, Theta, etc.). Nowadays, products such as variance swaps and volatility swaps and their variants allow for a pure volatility investment. Due to the increasingly important role of volatility, the first volatility index (Ticker:VXO) was launched by the Chicago Board Options Exchange (CBOE) in 1993. Since being revised to VIX in 2004, the index provides a quote of the fair fixed variance for a variance swap with a 30 day term for the S&P 500 index. In 2004, the VIX future was launched, followed by VIX options in 2006.

Both the VIX level and implied volatilities from options are often used as means to predict future fluctuations in the market. It is often said to be "the investor fear gauge" (Wahley, 2000), as the VIX index is often very quickly pushed up by investors' fear in the market. According to the statistics of CBOE, the past years in particular are marked by an increase in both trading volume and an outstanding amount of VIX futures. Not limited to the

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US market, individual investors can now easily trade volatility for most major equity indices (e.g. EuroStoxx, Nikkei, S&P 500) all over the world and at different term structures by trading options or futures on indices similar to the VIX. The sharp increase in trading volume of VIX futures observed during the subprime crisis reflects investors' intention to diversify in order to protect their investment.

There are further incentives of trading volatility. Unlike other assets, volatility generally shows a mean-reverting characteristic; it can grow neither to arbitrary high or low levels, but tends towards a medium level in the long run. It is well documented that volatility is negatively correlated with the stock index (Daigler and Rossi, 2006; Whaley, 2000; Giot, 2005). This negative correlation can be explained as the leverage hypothesis, where a decline in the equity level increases the leverage of the firm (market) and hence the risk to the stock (index). By studying the S&P 100 / S&P 500 indices and their volatility indices, Whaley (2000), Giot (2005) and Hibbert *et al*. (2008) point out the asymmetry of this negative relationship. While an increase in the VIX is marked by a drop in the stock market, a fall in the VIX is marked by only a small increase in the stock market.

In Figure 1, we calculate the rolling 1-year window correlation between the VIX and the S&P 500 index, noting that the correlation remains highly negative (-80%) during the crisis. Dash and Moran (2005) illustrates that implied volatility could provide a robust diversification method to reduce the investment risk.
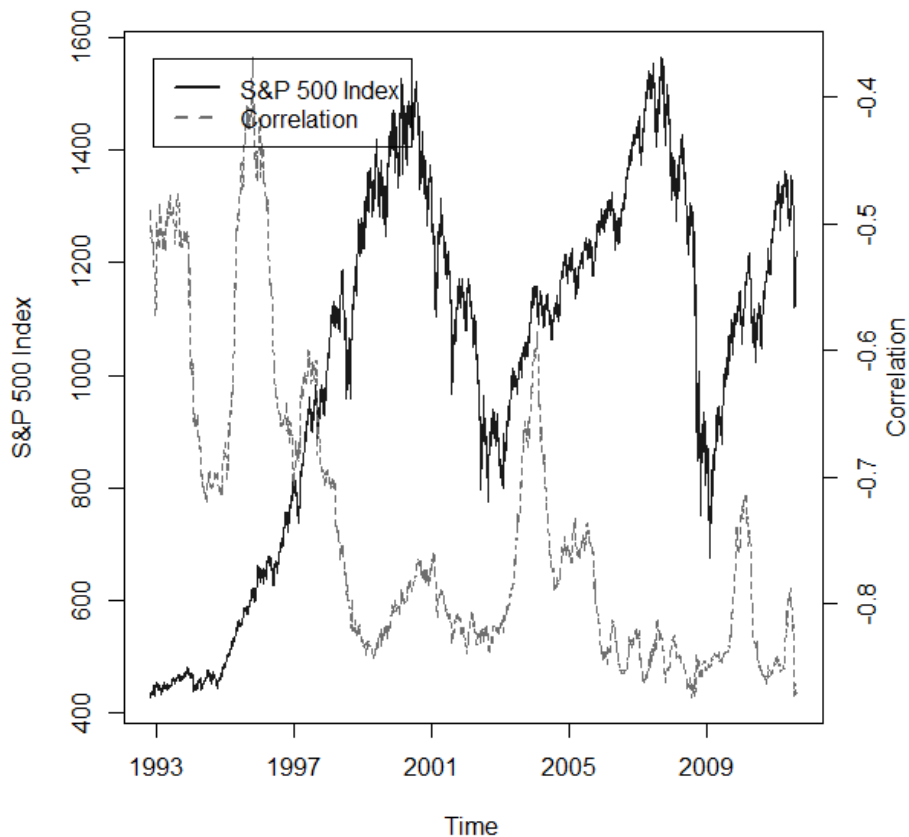

Fig. 1. Correlation between S&P 500 Index and VIX

Since the trading activity of volatility products increases, asset managers also use such products to optimize their investment portfolios. Carr and Madan (1998) provide a good overview of replicating variance swaps with options. Lee (2010) discusses pricing assuming no jumps. Bollerslev and Tauchen (2011) investigate jump tails and their compensation in the variance risk premium. Carr and Lee (2009) tackle the problem of replication errors of variance swaps. Todorov (2010) concludes that jump risk is priced in the market and investors' willingness to buy protection is sensitive to market jump events. During normal market phases (such as time horizon between 2004 and 2007 in Figure 2 and 3, implied short term volatility is often higher than realized volatility. This negative risk premium for implied variance has been studied in several articles such as Carr and Wu (2009) and Hafner and Wallmeier (2007). Carr and Wu (2009) also argue that an increase in market volatility is an unfavorable shock

to investors and that they would thus be willing to pay a protection premium. Other studies such as Bollerslev *et al.* (2009), Drechsler and Yaron (2011) establish the link between the variance swap premium and the equity premium.

Our estimations suggest that the average negative variance risk premium is around 3.5% in volatility for 30-day variance swap contracts, and 2.9% in volatility for 1-year variance swap contracts. Hafner and Wallmeier (2007) propose to add an exclusive variance swap selling strategy to a portfolio to earn such premium. However, a strategy based exclusively on selling does not make use of the negative correlation between the variance swaps and the underlying asset, thus rendering the diversification effect trivial. A long volatility strategy would significantly help investors to diversify and protect their portfolio during a financial crisis. However, this would be an expensive means of protection as it causes negative returns during normal market phases.
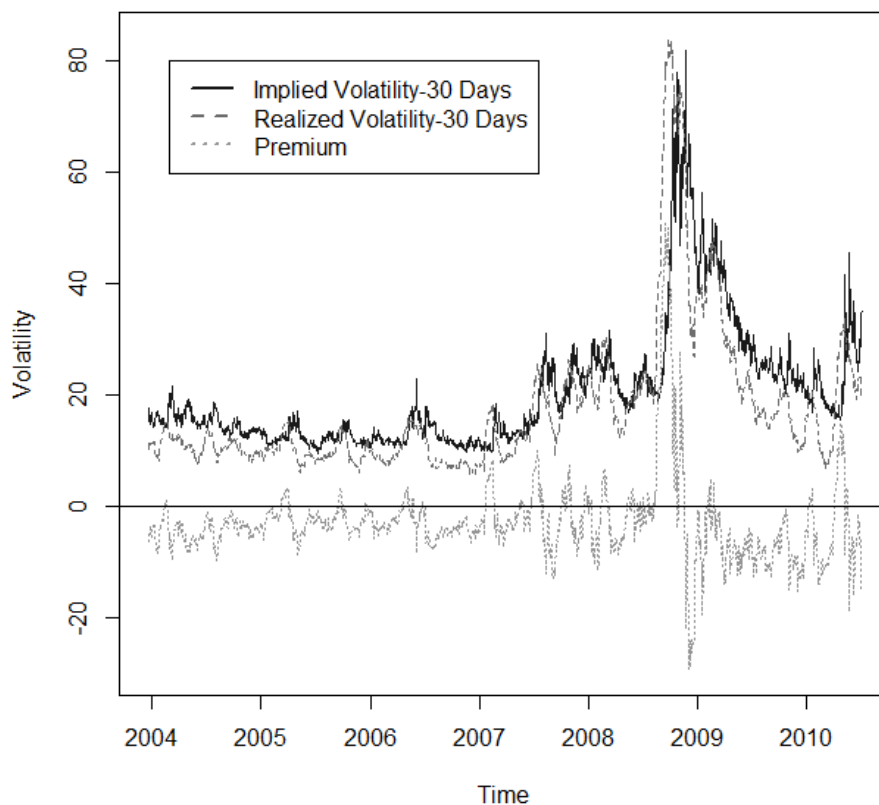
Fig. 2. Negative Risk Premium of 1-Month Implied Volatility

In this paper we complement existing research: we introduce a trading strategy of implied volatility that provides diversification at almost zero cost. We use the idea of calendar spreads from option trading and apply it to variance swaps. A calendar spread of variance swaps combines selling short term and buying long term variance swaps. This strategy can avoid the high costs encountered in single buying strategies and the low diversification effects in simple selling strategies. The key idea here lies in buying the variance swaps with long maturities at low frequencies while selling short term swaps at a higher frequency. At each trading point, we weigh the long term contract with a much higher notional than the short term ones in order to maintain a net zero *vega*. In the empirical analysis, we use original broker quotes for 1-, 3-, 6- and 12-month variance swaps between 2004 and 2011. We assume our original investment portfolio to consist of 70% S&P 500 and 30% fixed income investment. We apply a calendar spread of buying 1- year and selling 1-month variance swaps to diversify the portfolio. Our analysis shows that adding this strategy raises the mean return of the original portfolio by around 250%, while reducing the realized portfolio volatility to 50% of the original portfolio's volatility.

The paper is organized as follows: Section 2 briefly introduces the valuation model of variance swaps and Section 3 analyzes the performance of trading single variance swaps. In Section 4, the calendar spread settings are introduced and we present the performance of a predefined case. Further, we expand the strategy to different weights and illustrate more general cases of performance. We will show the robustness of calendar spread strategy in Section 5 and conclude in Section 6.
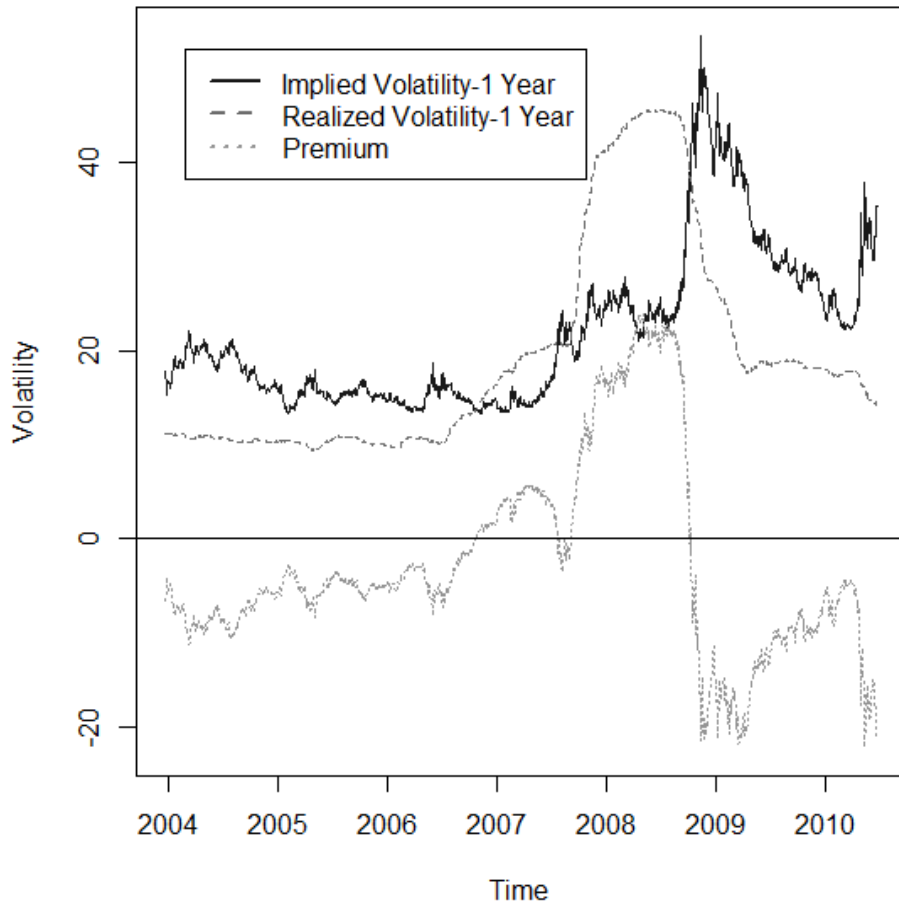
Fig. 3. Negative Risk Premium of 1-Year Implied Volatility

Empirical Study Setup

Variance swaps started trading in the late 1980s as an over-the-counter (OTC) derivative paying the difference between the realized variance σ_R^2 and the predefined implied variance or fixed variance σ_K^2. Usually the fixed variance σ_K^2 is determined such that the initial value of the swap is zero. The variance swap can be replicated by trading a series of out-of-the-money (OTM) call and put options. The term of a variance swap contract is taken to be 1-month (30-days) and 1-year (365-days) in our study. For simplicity, if the end of a variance swap is not a business day, the settlement of the variance swap contract is adjusted to be the last business day over the duration period of the study. Generally, variance swap contracts are traded in dollar notional (Notional). In our empirical analysis, we will use fix leg payments (or "DollarAmount") to measure the weights of the variance swaps. The relationship between notional and dollar amount is given by,

$$\text{DollarAmount} = \text{Notional} \cdot \sigma_K^2$$

We denote the vega exposure of variance swaps with maturity t as Vega_t. It is calculated by $\text{Vega}_t = 2 \cdot \text{Notional} \cdot \sigma_K$. The vega exposure will be positive if we buy implied volatility and negative if we sell implied volatility. The contract ends with a cash settlement at maturity, determined by the formula

$$\text{Payoff} = \text{Notional} \cdot (\sigma_R^2 - \sigma_K^2) \tag{1}$$

Here, the realized variance σ_R^2 is calculated by

$$\sigma_R^2 = \frac{252}{N} \sum_{i=1}^{N} (x_i)^2 \cdot 100 \tag{2}$$

Here, we use daily log returns $x_i = \ln \frac{S_i}{S_{i-1}}$ and denote by N the number of business days within the contract. The mark-to-market value of a variance swap at any time $t \in (0, T)$ is the weighted sum of realized variance and implied variance,

$$MTM_t = \text{Notional} \cdot (1 + r)^{-(T-t)} \cdot [\tfrac{t}{T}\sigma(0, t)_R^2 + \tfrac{T-t}{T}\sigma(t, T)_K^2 - \sigma_K^2] \tag{3}$$

Here, the $(1 + r)^{-(T-t)}$ is the present value at time t of receiving 1\$ at maturity T. The term $\sigma(0, t)_R^2$ denotes the realized variance until time t and $\sigma(t, T)_K^2$ the implied fair fixed variance of the variance swap from time t to Maturity. In our analysis, the value of $\sigma(t, T)_K^2$ is determined by linear interpolation. The inputs are fair fixed variances of variance swaps with maturity of 1 month, 3 months, 6 months and 1 year. We use the fair fixed variance for any variance swap that is shorter than 30 days and apply a linear interpolation to calculate $\sigma(t, T)_K^2$ in other cases as

$$\sigma(t, T)_K^2 = \tfrac{T-t1}{t2-t1}\sigma(t, t1)_K^2 + \tfrac{t2-T}{t2-t1}\sigma(t, t2)_K^2, \tag{4}$$

where t1, t2 are the limits of the time interval containing T and $\sigma(t, t1)_K^2$, $\sigma(t, t2)_K^2$ are the corresponding fair fixed variances of the variance swaps. The vega exposure of a variance swap decays linearly with time, which reduces the sensitivity of the value of the variance swap with respect to the change of implied volatility,

$$Vega_t = \frac{\partial MTM_t}{\partial \sigma(t,T)_K} = \text{Notional} \cdot (1 + r)^{-(T-t)} \cdot (2 \cdot \sigma(t, T)_K) \cdot \tfrac{T-t}{T} \tag{5}$$

Single Variance Swap Strategy

We assume the underlying portfolio to be a mix of 70% of S&P 500 index and 30% fixed income investment (70-30 portfolio) with an initial value of \$1 Million. In the first part of our empirical analysis we will demonstrate the diversification effect by adding a single variance swap to this 70-30 portfolio. We consider the time horizon from January 2004 to June 2011, where the sub-prime crisis is covered. Further, we focus on two contracts, the short term (1-month) and long term (12-month) variance swaps. We use the daily log change of the stock index and implied volatility and list basic statistics concerning their features in Table 1. One may find that both indices show non-normal characteristics due to their high excess kurtosis. We expect an extreme fat tail in the distribution of implied volatility.

Table 1. Summary Statistics

|	*Mean*	*Volatility*	*Skewness*	*Kurtosis*
VarSwap 1M	0.00% (-0.46%p.a.)	6.59% (104.68% p.a.)	0.59	4.03
VarSwap 1Y	0.02% (4.37%p.a.)	2.43% (38.61% p.a.)	0.53	4.38

In general, there are four scenarios in which a single variance swap can be added to a portfolio - selling or buying 1-month/1-year variance swaps. In our analysis, we assume no transaction costs and the portfolio to be rebalanced at a weekly frequency. From Equation (5), we notice that the vega exposure of a variance swap decays linearly with time. In order to maintain a more stable vega for the strategy, we need to trade a variance swap at a frequency that allows the maturities of the contracts to overlap. Considering the possible transaction costs in practice, we use four contracts overlapping each other at any valuation point. For a 1-month variance swap, we assume it to be traded weekly on each Friday or last business day of any week. The trading frequency for a 1-year contract is quarterly on the first business day of January, April, July and October. Another important issue is how to manage the notional of the variance swap. If we measure the weights by notional, the vega exposure will have a positive relation with the prevailing implied volatility. This in turn would cause more vega risk in turbulent markets than at any other time. In general, a high risk exposure during the financial crisis is not a preferable situation for the investor. Consequently, we propose to use a dollar amount (Equation 1) to manage the notional. When we fix the total dollar amount, we will trade much more notional of variance swap during normal market phases than during turbulence. This dynamic notional management is in line with most investors' preference. Considering the portfolio value of 1 million, we set the dollar amount to be 7,500\$ for the 1-month variance swap at each trading point and

50,000$ for the 1-year variance swap. We abbreviate this type of notional management by "DN" in the remainder of the paper. In order to measure the portfolio performance as well as the potential risks, we use three risk adjusted measures: the Sharpe ratio, Kappa 1 (known as Sortino ratio) and Kappa 2 (known as Omega ratio). The results of different portfolios are presented in Table 2 and the cumulative returns are shown in Figure 4.

Table 2. Single Variance Swap Strategy

	Mean Return	Volatility	Sharpe Ratio	Kappa 1	Kappa 2
70/30 Strategy	3.72%	14.70%	0.1476	14.03	5.55
Selling VarSwap 1M	9.49%	17.98%	0.4417	29.17	10.74
Buying VarSwap 1M	-6.65%	12.26%	-0.6686	-25.48	-14.29
Selling VarSwap 1Y	4.18%	45.35%	0.0579	7.85	1.91
Buying VarSwap 1Y	3.25%	10.24%	0.1656	17.72	8.42

With the selling strategy, we realized similar results as Hafner and Wallmeier (2007) and Grant *et al*. (2007). The risk premium earned from the selling strategy significantly increases the portfolio's mean return. Through the weekly selling of a 1-month variance swap, the portfolio's mean return rises by more than 250% while the volatility only increases by less than 25%. Therefore, we observe much better risk-adjusted performance measures for the portfolio. However, when we look at the cumulative return in Figure 5 between 2008 and 2009, rather than provide protection during the crisis, selling variance swaps further lowers the portfolio's return. This does not favor investors looking to diversify with investments in volatility. Considering the buying strategy, the volatility of the new portfolio is considerably lower than the original one. During the financial crisis, the variance swap provides protection to the investor and we do not find a substantial drop in the total portfolio returns in the buying strategy. However, a high risk premium paid to protect the market from crashing sharply results in a reduction of the mean return from the investment. The risk premiums of buying variance swaps are even higher than the total returns from the index. In the case of buying a 1-month contract we even observe a highly negative return at 6.65% per year.
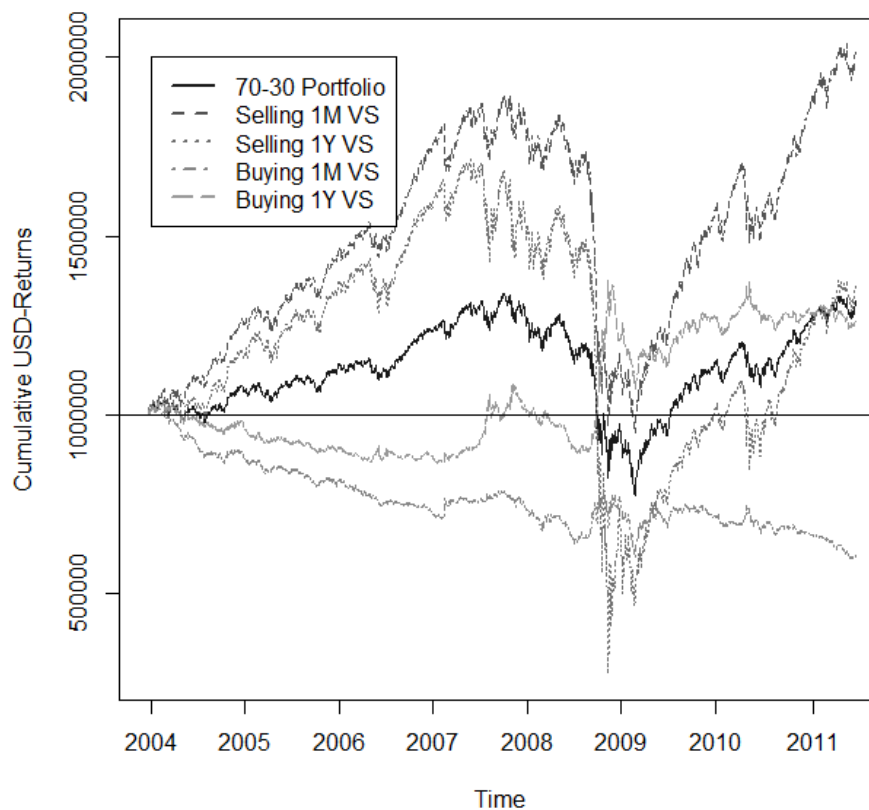
Fig. 4. Single Variance Swap Strategy

Calendar Spread Strategy

The shortcomings of single variance swap strategies are obvious. The selling strategy does not benefit from the diversification while the buying strategy is too expensive to be carried out in a normal market phase. We estimate that the average negative risk premiums of 1-month and 1-year variance swaps are 51.65 and 46.06 per notional. We can consider the expected marginal cost of buying 1 notional variance swap to be approximately 50$. Suppose we want to protect the portfolio against a sudden crash, we need to maintain a positive vega in our variance swap strategy. Therefore, a long variance swap would be our only choice. This would be expensive and we want to reduce the risk premium. It is not difficult to find that we can sell the variance swaps at the same time to earn a similar amount of risk premium as compensation. Further, the correlation between the movement of 1-month and 1-year implied volatility is strong at 84.56%. Hence, we expect a similar implied volatility term structure movement from this high correlation. In practice, a derivatives trader uses a calendar spread to benefit from this parallel term structure movement. In the simplest case, we buy twelve 1-year variance swaps and sell a 1-month contract with 1 notional each day. The expected risk premium to be paid on twelve 1-year swap contracts could be compensated by selling the 1-month swap. Thus, by using a calendar spread structure, we generate a strategy with almost zero expected risk premium. After one month the sold contract will expire and the long 1-year variance swap provides protection to the portfolio for the remaining eleven months. Within the first month, we hold both, the short and the long contract.

From Table 1, we know that the 1-month implied volatility is about 2.5 times the 1-year implied one. Therefore, if we want to offset the returns of a variance swap calendar spread, we need to keep the vega exposure for the long one to be 2.5 times higher than for the short one. It is not very difficult to solve this problem. We can expand the idea above to buying a 1-year variance swap today and selling a 1-month contract in the second month and repeat this for the third month. Now one could combine two simple strategies and buy 3 times the notional of the 1-year variance swaps at the beginning while selling a 1 notional swap on the first day of every month for the next three months. We can maintain the vega exposure for the long position to be around 2.5 times higher than for the short position. Finally, the expected risk premium of the long and short variance swaps offset each other and within the first three months, the risk of the two contracts would be well diversified and we get full protection for the remaining 9 months. We can repeat this strategy every quarter and the overlapping strategy would give reasonable protection to the portfolio.

Coming back to the example of the single strategy in the last section where 50,000 $ 1-year swaps are bought quarterly and 7,500$ 1-month variance swap contract are sold on a weekly basis. We want to present a more generalized case in this section. This arbitrary notional is selected to illustrate the effect of the calendar spread strategy. A more general performance analysis where the notional is determined as a percentage of portfolio value will be presented in Section 4. Theoretically, we have four contracts overlapping each other. We hold approximately 200,000$ in long variance swaps while holding 30,000$ in short variance swaps at any time. The vega difference between the long and short strategy is shown in Figure 5. If we now combine these two single strategies, the resulting strategy matches the key idea of a calendar spread for variance swaps – a positive vega exposure to the market. In each calendar year, we estimate to sell 360,000$ of variance swaps which is greater than the 200,000$ buying amount, which result in an earning premium situation. We show the results of the calendar spread variance swap strategy in Figure 5. As there is no initial investment for the variance swap, we add one million dollar cash as buffer, which bear no interest to the strategy, to calculate the performance of the calendar spread.

In Figure 6 we present the total returns of the exclusive short strategy, the exclusive long strategy and the calendar spread strategy. Prior to 2008, unlike the portfolio value of the exclusive long strategy, which falls below the initial value, the calendar spread actually generates positive returns. During the financial crisis in 2008 the calendar spread jumps upward while the value of the exclusive short strategy drops substantially. The calendar spread strategy reaches the highest risk adjusted performance in kappa 1 and kappa 2. Unlike the negative mean return of the 1-year strategy, it has a significant positive return of 5.43% in our case, which is only about 1% lower than the exclusive short strategy. Finally, we estimate that the correlation between calendar spread and our underlying portfolio is around 79.70%. Recognizing the advantages of the calendar spread, we now add it to the portfolio to analyze to which extent the performance can be improved. We present the cumulative returns of the combined portfolio in Figure 7 and summary statistics in Table 4.
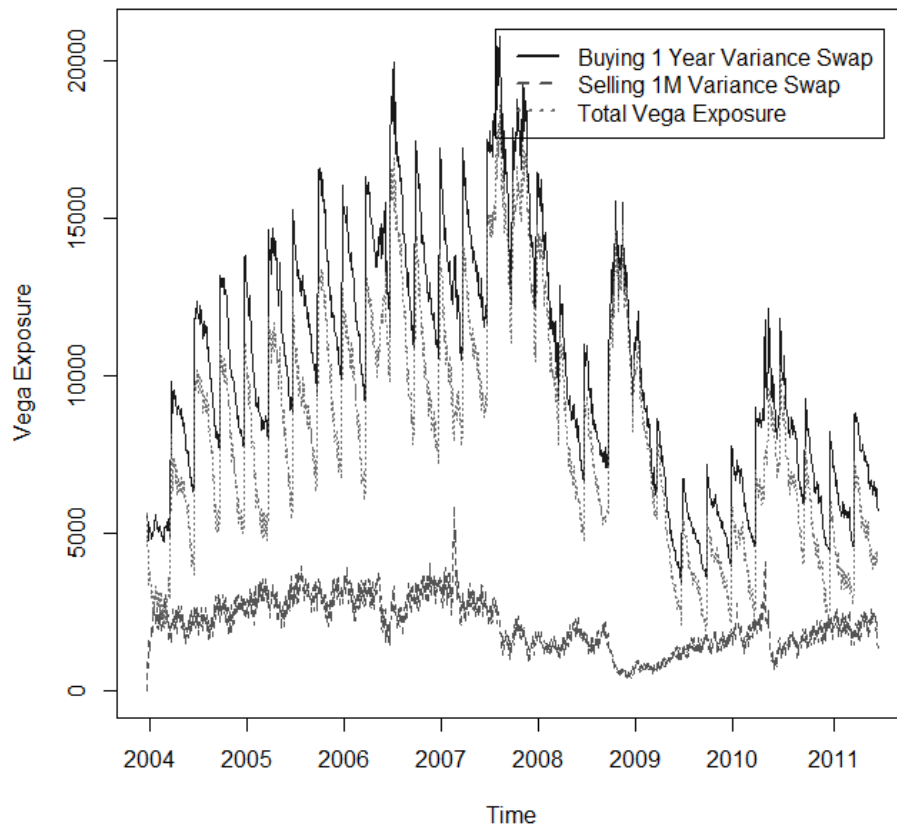
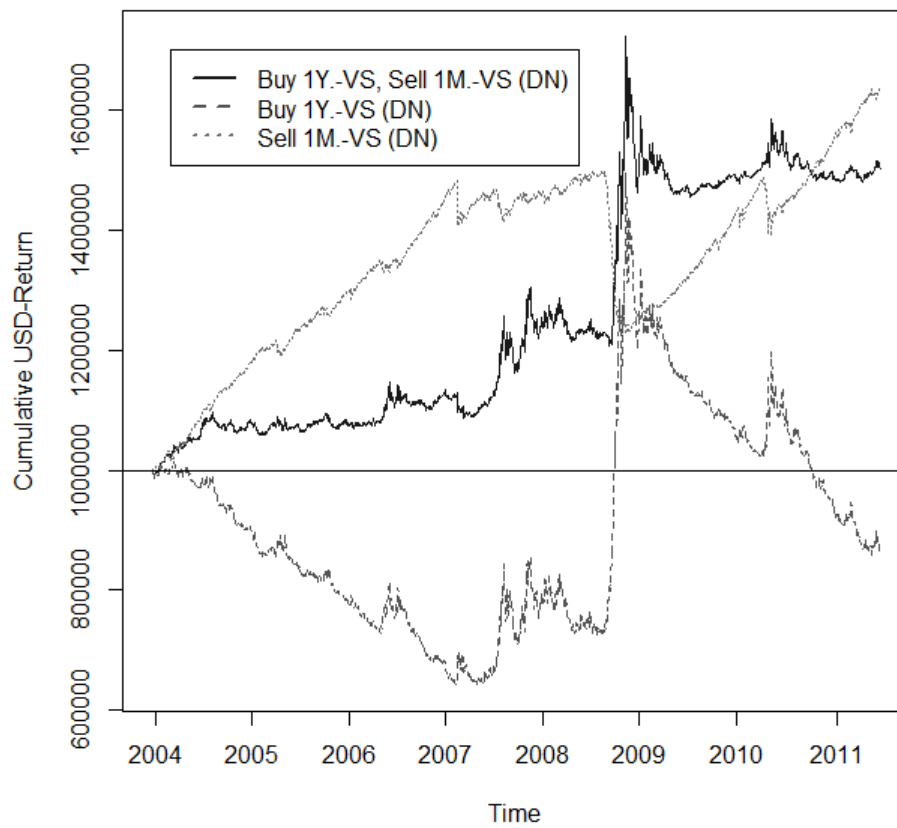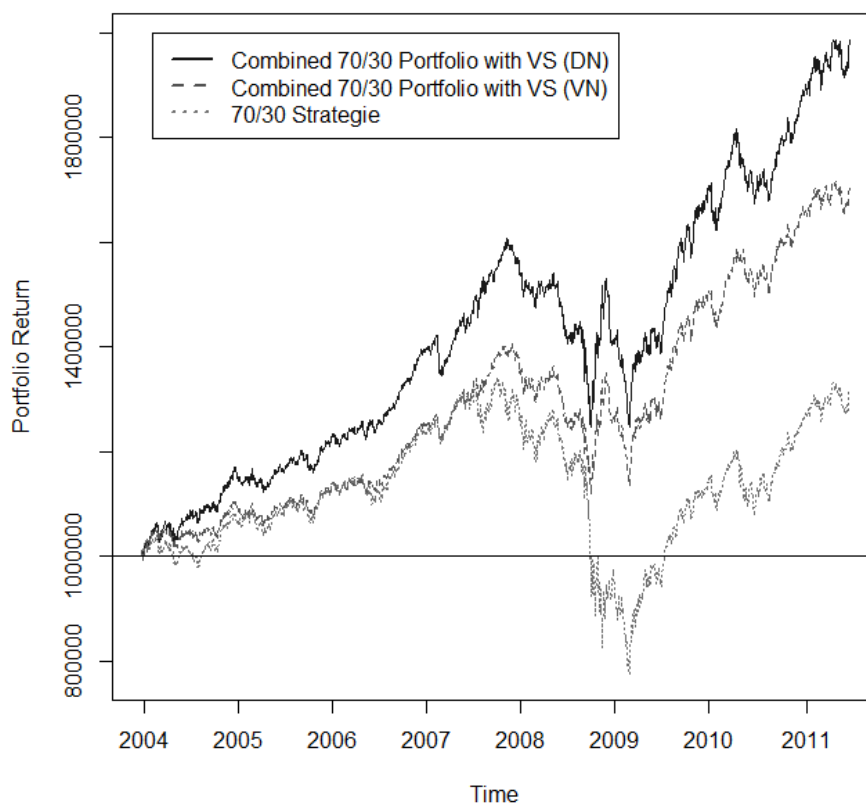**Fig. 5.** Vega Exposure in Calendar Spread Strategy

Fig. 6. Calendar Variance Swap Strategy

Fig. 7. Combined Performance with Calendar Spread of Variance Swaps ("VN" Refers to Vega-neutral Calendar Spreads of Variance Swaps)

Adding the calendar spread to the 70/30 portfolio increases the average return from 3.72% to 9.19% while reducing the volatility to 9.63%, which is 5% less than in the original portfolio. The combined portfolio shows a Sharpe ration 5 times higher than the original portfolio, and Kappa 1 and Kappa 2 measures 2-3 times greater as well. As we see from Figure 6, during 2008 the down side drop is considerably reduced by the variance swap. In Table 5 we show some risk measures at the tails. By incorporating a calendar spread, the minimum return and high quantile VaR are significantly reduced. We conclude that the calendar spread strategy using two variance swaps with different maturities correlates highly negatively with the stock index while no or even positive premium is earned.

Table 3. Buying 1-Year, Selling 1-Month Variance Swap Strategy

Strategy	Selling 1-Month	Buying 1-Year	Combined
Mean Return	6.59%	-1.70%	5.43%
Volatility	5.08%	21.61%	10.28%
Sharpe Ratio	1.2975	-0.0787	0.5284
Kappa 1	29.17	17.72	49.00
Kappa 2	10.74	8.42	20.76

Taking this idea further, we will weigh a 1-year and a 1-month variance swap differently using the calendar spread strategy. As we know, the portfolio value changes with market fluctuation. Therefore, a fixed notional would not be a good choice for portfolio management. We adjust the notional when buying a 1-year contract to be a percentage of the total portfolio value. As we sell a 1-month contract at a smaller amount but at a higher frequency each time, we weigh the dollar amount sold as a proportion of the 1-year variance swap notional. We set the minimum amount of buying a 1-year variance swap to be 1% of the total portfolio value and cap the maximum amount at 10%. The 1-month variance swap would have a weight of 5% compared to 10% of the 1-year notional.

Table 4. Statistics of Portfolios

|	*S&P 500*	*JPM*	*70/30 Strategy*	*Combined (DN)*
Mean Return	2.52%	5.29%	3.72%	9.19%
Volatility	21.40%	4.07%	14.70%	9.63%
Sharpe Ratio	0.0441	0.9178	0.1475	0.7924
Kappa 1			14.00	49.00
Kappa 2			5.55	20.76

Table 5. Risk Measures of Portfolios

	Min	*VaR 99%*	*VaR 99.9%*
70/30 Strategy	6.54%	6.21%	2.90%
Combined	5.36%	3.58%	1.78%

All results are presented graphically in Figures 8 and 9. If we fix the weights of the 1-month notional, we see that the Sharpe ratio is a concave function of the 1-year notional. If we only buy a small amount of the 1-year variance swaps, it will not provide enough protection during a crisis. However, if the weight of a 1-year contract is too high, the portfolio would not perform well because of the high risk premium paid when buying variance swaps. It is thus very important to determine suitable weights for the 1-year variance swap. The portfolio volatility is a convex function of the weights of a 1-year contract. In summary, weighting 1-year variance swaps at 3%-4% of the total portfolio value would most significantly diversify the risk in the portfolio and can hence be considered a preferred choice, as this strategy provides both a high Sharpe ratio and the desired diversification effect.

If we fix the amount of long 1-year contracts and sell a larger number of 1-month contracts, this would always generate a positive marginal change of the portfolio Sharpe ratio. However, the speed at which it increases would decrease if we weigh them at more than 20% of the long amount. The volatility of the portfolio does not change significantly when we raise the weights of the sold notional. Let us consider the effect on the Sharpe ratio. As we stated before, selling single variance swaps would increase the Sharpe ratio but aggregate the loss in case of a financial crisis. This is why it is important to determine the most suitable ratio. Figure 9 shows that we can expect a 1-year notional in the range of 15%-20% to yield the best portfolio performance.
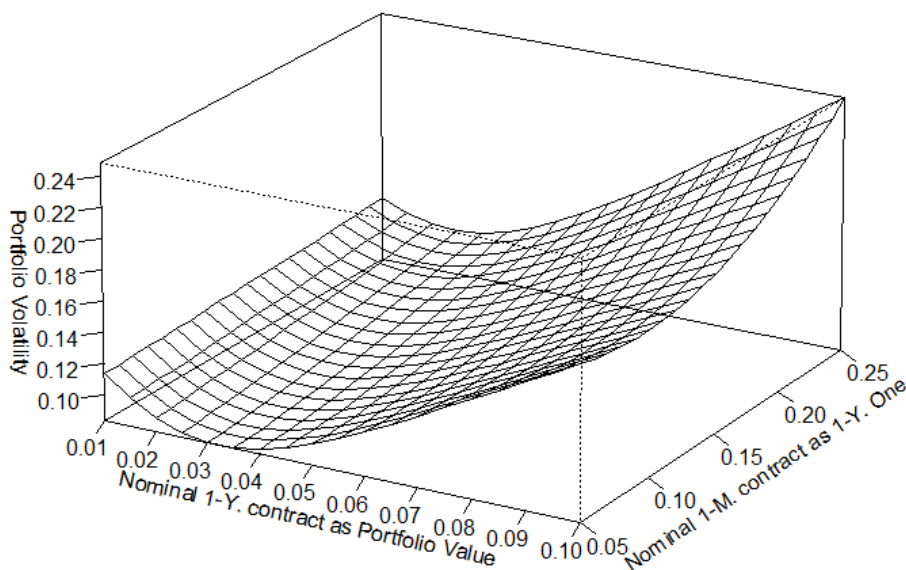
Fig. 8. Portfolio Volatility for Different Weight Schemes of Variance Swaps
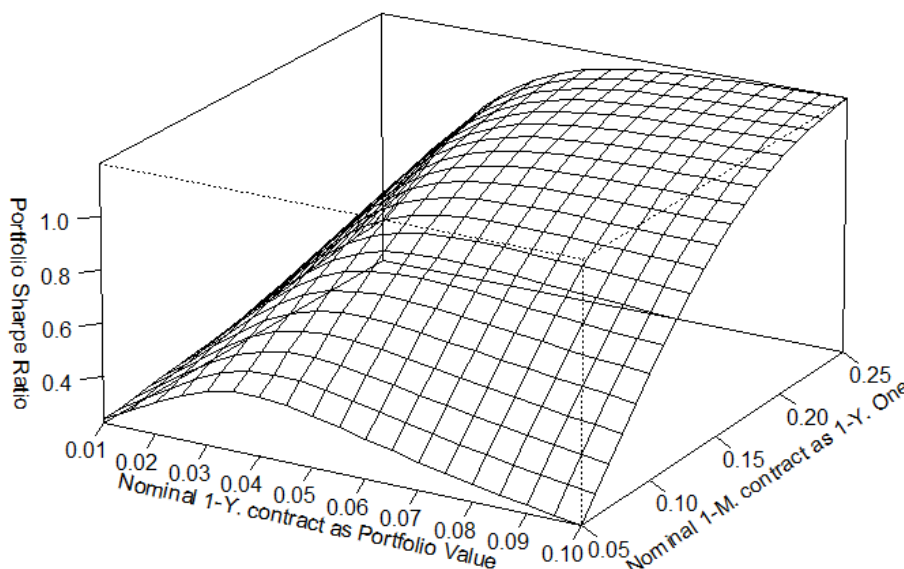
Fig. 9. Portfolio Sharpe Ratio for Different Weight Schemes of Variance Swaps

Robustness of the Strategy

So far, we have observed the significant performance improvement achieved through trading a calendar spread variance swap strategy. However, we assume that the trading occurs on some specific dates. One may argue that the strategy may not work well if we shift the trading date. We now want to show the robustness of this calendar spread strategy in the scenario of a change in dates. We maintain the same trading frequency, but choose a different starting date. For the single 1-month strategy, we will show the results if we trade from every Monday to Friday and for selling the 1-year strategy, we shift the starting date by five business days (weekly). If a given date is not a business day, we shift them to the day after.

All the results are presented in Table 6 and Table 7. For the selling 1-month strategy, we always reach a Sharpe ratio greater than 0.8, and in most cases, a Sharpe ratio greater than 1. Similarly, the Sharpe ratio of selling 1-year contracts is always positive. We consider the difference between different Sharpe ratios within an acceptable range. We also get similar results when shifting trading days of variance swaps on the EuroStoxx index.

Table 6. Trading Day Shift of the 1-Month Strategy

Day	Mo	Tu	We	Th	Fr
Mean Return p.a.	4.78%	4.34%	5.05%	5.46%	5.69%
Sharpe Ratio p.a.	1.0154	0.8868	1.1158	1.2181	1.2420
Difference in %	0.00%	-19.78%	0.94%	10.20%	12.36%

Table 7. Trading Day Shift of the 1-Year Strategy

Week	0	1	2	3	4	5
Mean Return p.a.	2.15%	2.35%	2.35%	2.08%	2.05%	1.86%
Sharpe Ratio p.a.	0.1069	0.1269	0.1324	0.1078	0.1090	0.0921
Difference in %	0.00%	18.78%	23.89%	0.86%	1.99%	-13.80%
Week	6	7	8	9	10	11
Mean Return (USD)	1.70%	1.75%	2.02%	1.57%	1.80%	1.92%
Sharpe Ratio p.a.	0.0837	0.0868	0.0999	0.0720	0.0874	0.0895
Difference in %	-21.66%	-18.79%	-6.54%	-32.66%	-18.18%	-16.28%

Week	12					
Mean Return (USD)	2.42%					
Sharpe Ratio p.a.	0.1220					
Difference in %	14.17%					

Conclusions

In this paper, we show the impact of variance swap based strategies on the long-term performance of a stock index portfolio. We confirm the findings in existing literature in our back-testing analysis of the S&P500 and EUROSTOXX50 index: we find that, by selling single variance swaps, one can improve the Sharpe ratio of the portfolio by achieving a much higher mean return. In the post crisis time, the implied volatility remains at a high level while the market is recovering and has a much smaller realized volatility. However, a pure selling strategy would also leverage the loss during the financial crisis and would not help diversifying the risk in the portfolio. We analyze a calendar spread strategy that consists of buying long term variance swaps and selling short term variance swaps at a higher frequency.

During back-testing, we construct a rolling calendar spread strategy trading variance swaps to avoid the shortcomings encountered when trading single variance swap contracts. Ignoring bid-offer spreads, this calendar spread strategy yields reasonable protection from potential market crashes at almost zero or even negative costs. The combined strategy can maintain a correlation of almost -80% to the original portfolio, providing a noticeable diversification effect. We show that the volatility of a 70/30 equity-bond portfolio would decrease from 14.70% to 9.63% while the mean return would increase from 3.72% to 9.19%. Besides that, the calendar spread also greatly reduces the tail risk at a higher quantile. The potential loss at the 0.1% and 1% quantile would be less than 60% of the original portfolio. Last, we link the notional of a variance swap to the portfolio value. We suggest that buying 1-year variance swaps at 3-4% of the total value and selling 1-month variance swaps at 15-20% to the 1-year notional would provide the best performance of the calendar spread strategy.

Finally, we would like to point out that the results are based on a back-testing analysis and have no predicative power. For example, the strategy may perform badly in the future in case of a linear (zero variance) asset melt-down.

References

Bollerslev, T. and Tauchen, G. (2011). Tails, Fears, and Risk Premia. The Journal of Finance, 66, 2165-2211.

Bollerslev, T., Tauchen, G. and Zhou, H. (2009). Expected Stock Returns and Variance Risk Premia. Review of Financial Studies, 22(11), 4463-4492.

Carr, P. and Lee, R. (2009). Volatility Derivatives. Annual Review of Financial Economics, 1, 319-339.

Carr, P. and Madan, D. (2001). Towards a Theory of Volatility Trading. Option Pricing, Interest Rates and Risk Management, Cambridge University Press, 458-476.

Carr, P. and Wu, L. (2009). Variance Risk Premiums. Review of Financial Studies, 22(3), 1311-1341.

Daigler, R. T. and Rossi, L. (2006). A Portfolio of Stocks and Volatility. The Journal of Investing, 15, 99-106.

Dash, S. and Moran, M. T. (2005). VIX as a Companion For Hedge Fund Portfolios. The Journal of Alternative Investments, 8, 75-80.

Drechsler, I. and Yaron, A. (2011). What's Vol Got to Do with It. Review of Financial Studies, 24(1), 1-45.

Giot, P. (2005). Relationships between Implied Volatility Indexes and Stock Index Returns. The Journal of Portfolio Management, 31(3), 92-100.

Hafner, R. and Wallmeier, M. (2007). Volatility as an asset class: European evidence. European Journal of Finance, 13(7), 621-644.

Hibbert, A. M., Daigler, R. T., and Dupoyet, B. (2008). A Behavioral Explanation for the Negative Asymmetric Return-Volatility Relation. Journal of Banking & Finance, 32(10), 2254-2266.

Todorov, V. (2010). Variance Risk-Premium Dynamics: The Role of Jumps. Review of Financial Studies, 23(1), 345-383.

Whaley, R. E. (2000). The Investor Fear Gauge. Journal of Portfolio Management, 26, 12-17.