On the Cost of Delayed Currency Fixing Announcements

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Abstract

In Foreign Exchange Markets vanilla and barrier options are traded frequently. The market standard is a cutoff time of 10:00 a.m. in New York for the strike of vanillas and a knock-out event based on a continuously observed barrier in the inter bank market. However, many clients, particularly from Italy, prefer the cutoff and knock-out event to be based on the fixing published by the European Central Bank on the Reuters Page ECB37. These barrier options are called discretely monitored barrier options. While these options can be priced in several models by various techniques, the ECB source of the fixing causes two problems. First of all, it is not tradable, and secondly it is published with a delay of about 10 - 20 minutes. We examine here the effect of these problems on the hedge of those options and consequently suggest a cost based on the additional uncertainty encountered.

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1 Overview

1.1 Agenda

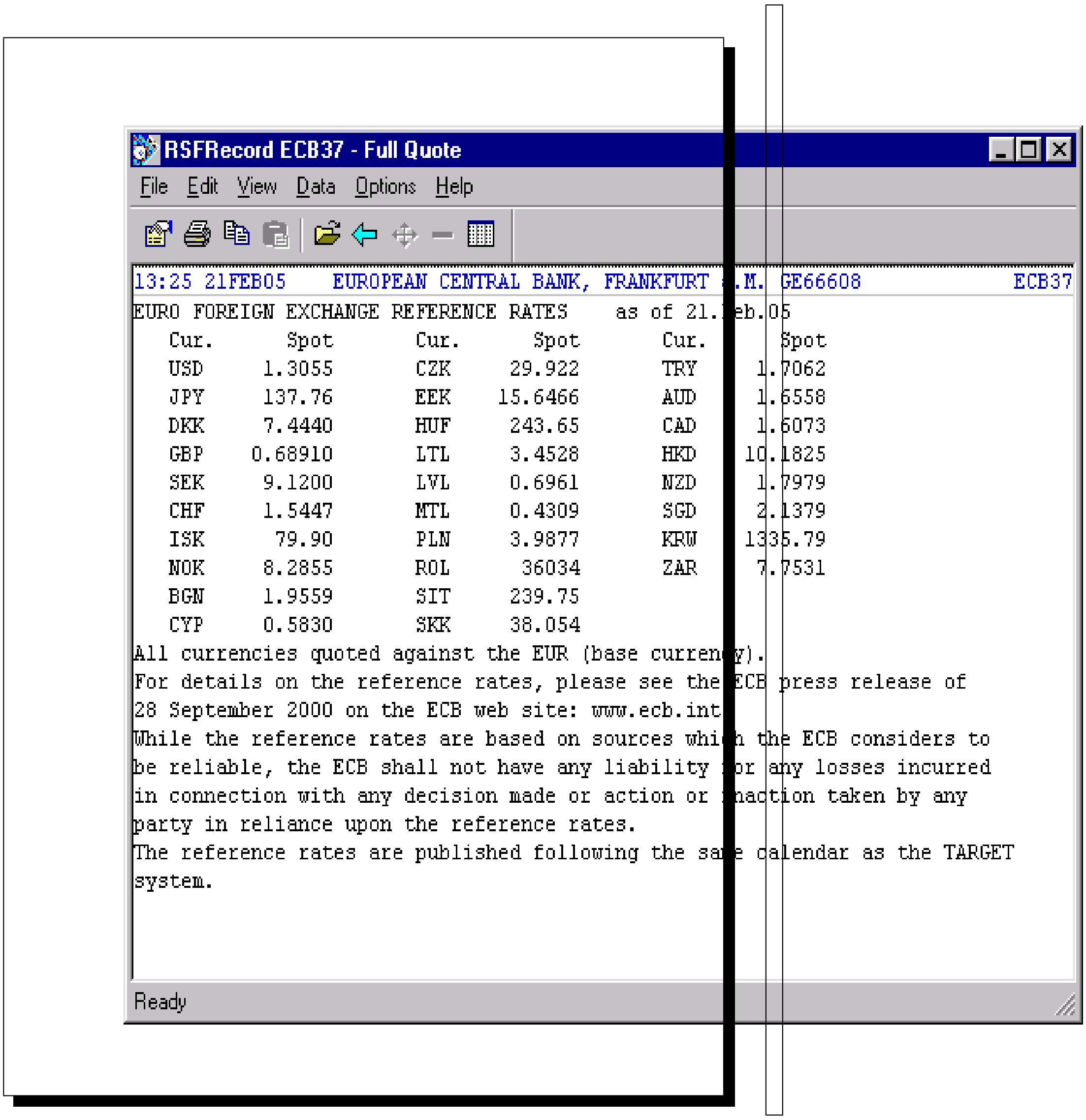
1. Take a liquid currency pair such as EUR-USD
2. Consider Delta-Hedging a short position of
3. A European style Reverse Knock-Out (RKO) Call
4. Or a discretely monitored RKO
5. Analyze cost due to unknown spot value at knock-out or expiration

1.2 Cut-Off Time and Value

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
- FX: OTC standard is NY cut: Traded FX spot at 10:00 a.m. NY time
- Problem: not official, not transparent to public
- Advantage: tradable, transparent to FX traders
- Other sources: FED, Warshaw Cut, Tokio Cut, Bank's own fixing
- Average of several banks
- Example: OPTREF = AVG (COMBA, DB, DREBA, HVB)

1.3 ECB currency fixing

1. Many Corporate Treasurers prefer official source of exchange rate
2. Set each business day at 2:15 p.m.
3. Published on Reuters page ECB37
4. Not tradable
5. Published with Delay of ΔT = 10-20 Minutes


RSFRecord ECB37 - Full Quote

File Edit View Data Options Help

13:25 21FEB05 EUROPEAN CENTRAL BANK, FRANKFURT A.M. GE66608 ECB37

EURO FOREIGN EXCHANGE REFERENCE RATES as of 21.Feb.05

Cur.	Spot	Cur.	Spot	Cur.	Spot
USD	1.3055	CZK	29.922	TRY	1.7062
JPY	137.76	EEK	15.6466	AUD	1.6558
DKK	7.4440	HUF	243.65	CAD	1.6073
GBP	0.68910	LTL	3.4528	HKD	10.1825
SEK	9.1200	LVL	0.6961	NZD	1.7979
CHF	1.5447	MTL	0.4309	SGD	2.1379
ISK	79.90	PLN	3.9877	KRW	1335.79
NOK	8.2855	ROL	36034	ZAR	7.7531
BGN	1.9559	SIT	239.75		
CYP	0.5830	SKK	38.054		

All currencies quoted against the EUR (base currency).
For details on the reference rates, please see the ECB press release of
28 September 2000 on the ECB web site: www.ecb.int
While the reference rates are based on sources which the ECB considers to
be reliable, the ECB shall not have any liability for any losses incurred
in connection with any decision made or action or inaction taken by any
party in reliance upon the reference rates.
The reference rates are published following the same calendar as the TARGET
system.

Ready

1.4 Model

Risk-neutral geometric Brownian motion

$$dS_t = S_t[(r_d - r_f)\,dt + \sigma\,dW_t]$$

These parameters are constant

- r_d: domestic interest rate
- r_f: foreign interest rate
- σ: volatility
- S_t: FX spot rate at time t

1.5 Contracts

- T: maturity in years
- K: strike
- B: knock-out barrier
- fixing schedule $0 = t_0 < t_1 < t_2 \ldots, t_n = T$
- payoffs for the vanilla and for a discretely monitored up-and-out call option

$$\begin{aligned} V(F_T, T) &= (F_T - K)^+ \\ V(F, T) &= (F_T - K)^+ I\!I_{\{\max(F_{t_0}, \ldots, F_{t_n}) < B\}} \end{aligned}$$

- F_t: fixing of the underlying exchange rate at time t
- $I\!I$: the indicator function
- payoffs to hedge are

$$\begin{aligned} V(S_T, T) &= (S_T - K)^+ \\ V(S, T) &= (S_T - K)^+ I\!I_{\{\max(S_{t_0}, \ldots, S_{t_n}) < B\}} \end{aligned}$$

1.6 Analysis Procedure

- simulate the spot with Monte Carlo
- model the ECB-fixing F_t by

$$F_t = S_t + \varphi, \quad \varphi \in \mathcal{N}(\mu, \sigma)$$

- μ and σ estimated from historic data.
- difference of fixing and traded spot = normally distributed random variable.

1.6.1 Estimates

Estimated values for mean and standard-deviations of the quantity Spot - ECB-fixing from historic time series. Data provided by Commerzbank.

Currency pair	Mean	Std Dev	Time horizon
EUR / USD	-3.125E-6	0.0001264	23.6 - 08.8.04
USD / JPY	-4.883E-3	0.0134583	22.6 - 26.8.04
USD / CHF	-1.424E-5	0.0001677	11.5 - 26.8.04
EUR / GBP	-1.330E-5	0.00009017	04.5 - 26.8.04

For USD-JPY take EUR-JPY / EUR-USD etc.

2 Error Estimation

1. introduce a bid/offer-spread δ for the spot, which is of the size of 2 pips in the inter bank market.
2. evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold.
3. compute for each path the error encountered due the fixing being different from the spot.
4. average over all paths.
5. do this for various currency pairs, parameter scenarios, varying the rates, volatilities, maturities, barriers and strikes.
6. We expect a significant impact particularly for reverse knock-out barrier options due to the jump of the payoff and hence the large delta hedge quantity.

2.1 European up-and-out Call

Error per 1 unit of the underlying. Three cases:

$S_T \leq K$

- the seller who is short the option decides not to hedge as the option is probably out of the money, i.e. delta = 0.
- if the option turns out to be in the money, i.e. $F_T > K$, the holder of the short position faces a P&L of $K - (S(T + \Delta T) + \delta)$ (units of the base currency).

$S_T > K$ **and** $S_T < B$

- one assumes that the option is in the money and delta is 1.
- if now $F_T \leq K$ or $F_T \geq B$, there is a P&L of $S(T + \Delta T) - (S(T) + \delta)$.

$S_T \geq B$ **and** $F_T < B$

- here we have a P&L of $K - (S(T + \Delta T) + \delta)$.
- note that other than in the first case, this P&L is of order $K - B$ due to the jump in the payoff.

2.2 Discretely monitored up-and-out Call

$S_t < B$ **and** $F_t \geq B$

- here we unwind our hedge with delay and encounter

$$\text{P\&L} = \Delta(S_t) \cdot (S_{t+\Delta T} - S_t),$$

- $\Delta(S_t)$: theoretical delta (negative near B).
- the seller has been short the underlying at time t and must buy it in $t + \Delta T$ minutes to close out the hedge.
- he makes profit if the underlying is cheaper in $t + \Delta T$.

$S_t \geq B$ **and** $F_t < B$

- here the seller closed out the hedge at time t, though she shouldn't have done so
- and in $t + \Delta T$ she needs to build a new hedge causing

$$\text{P\&L} = \Delta(S_t) \cdot (S_t + \delta) - \Delta(S_{t+\Delta T}) \cdot S_{t+\Delta T}$$

2.3 Calculating the Delta-Hedge Quantity

- approximation by Per Hörfelt in [5]
- Assume the value of the spot is observed at times iT/n, $i = 0, \ldots, n$
- define

$$
\begin{aligned}
\theta_\pm &\triangleq \frac{r_d - r_f \pm \sigma^2/2}{\sigma}\sqrt{T} \\
c &\triangleq \frac{\ln(K/S_0)}{\sigma\sqrt{T}} \\
d &\triangleq \frac{\ln(B/S_0)}{\sigma\sqrt{T}} \\
\beta &\triangleq -\zeta(1/2)/\sqrt{(2\pi)} \approx 0.5826
\end{aligned}
$$

- ζ: Riemann zeta function

- define

$$F_+(a, b; \theta) \triangleq \mathcal{N}(a - \theta) - e^{2b\theta}\mathcal{N}(a - 2b - \theta)$$

- obtain for the value of the discretely monitored up-and-out call

$$
\begin{aligned}
V(S_0, 0) \approx\ & S_0 e^{-r_f T}\left[F_+(d, d + \beta/\sqrt{n}; \theta_+) - F_+(c, d + \beta/\sqrt{n}; \theta_+)\right] \\
& -K e^{-r_d T}\left[F_+(d, d + \beta/\sqrt{n}; \theta_-) - F_+(c, d + \beta/\sqrt{n}; \theta_-)\right]
\end{aligned}
$$

- take a finite difference approach for the computation of the theoretical delta

$$\Delta = V_S(S, t) \approx \frac{V(S + \epsilon, t) - V(S - \epsilon, t)}{2\epsilon}$$

3 Analysis of EUR-USD

Spot	1.2100
Strike	1.1800
Trading days	250
domestic interest rate	2.17% (USD)
Foreign interest rate	2.27% (EUR)
Volatility	10.4%
Time to maturity	1 year
Notional	1,000,000 EUR

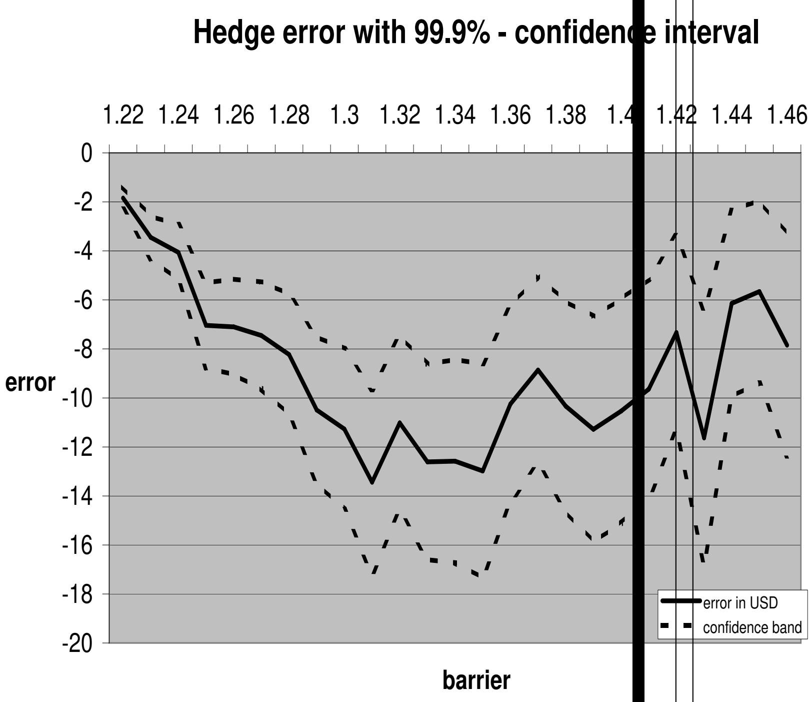
Consider a short position of a discretely monitored up-and-out Call

3.1 Distribution of Absolute Errors in USD

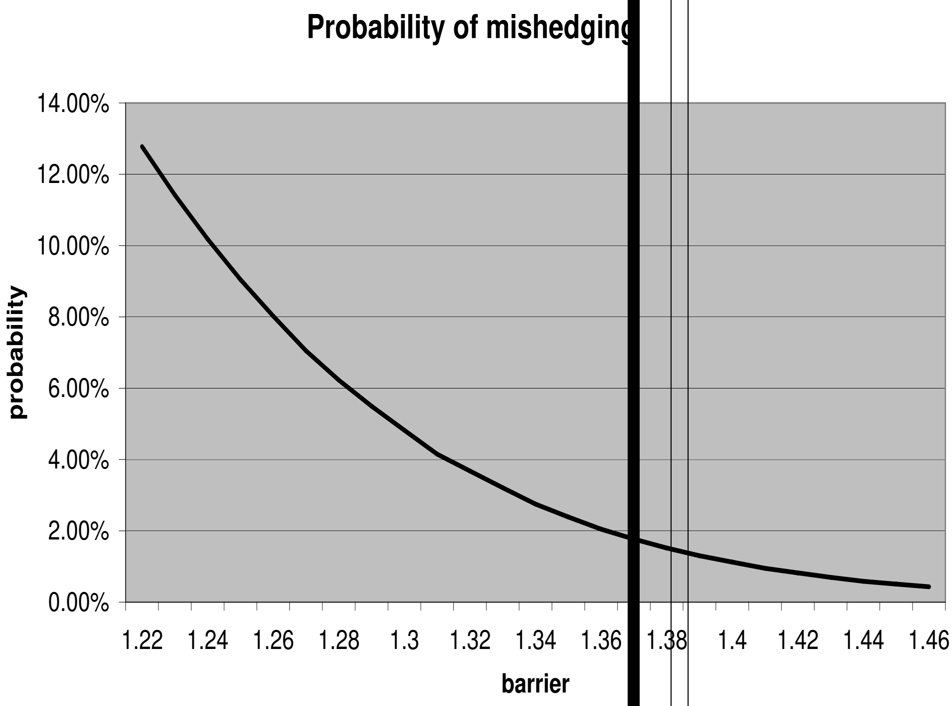
The figures are the number of occurrences out of 1 million

barrier 1.2500	<1k$	<2k$	<3k$	<39k$	<40k$	<[illegible]1k$	<42k$	<43k$
upside error	951744	20	1	0	0	0	0	0
downside error	48008	54	2	5	59	85	21	1
barrier 1.3000	<1k$	< 2k$	< 3k$	<89k$	< 90k$	<[illegible]1k$	<92k$	
upside error	974340	20	1	0	0	0	0	
downside error	25475	43	0	2	40	59	20	
barrier 1.4100	<1k$	<2k$	<3k$	<199k$	<200k$	<2[illegible]1k$	<202k$	<203k$
upside error	994854	78	0	0	0	0	0	0
downside error	4825	194	3	1	19	17	8	1

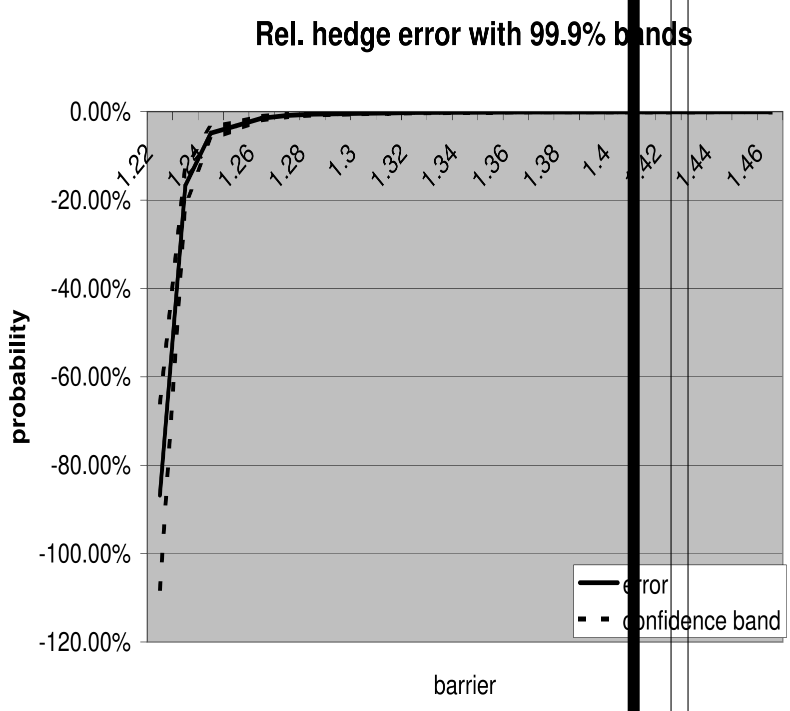
3.2 Additional Hedge Cost

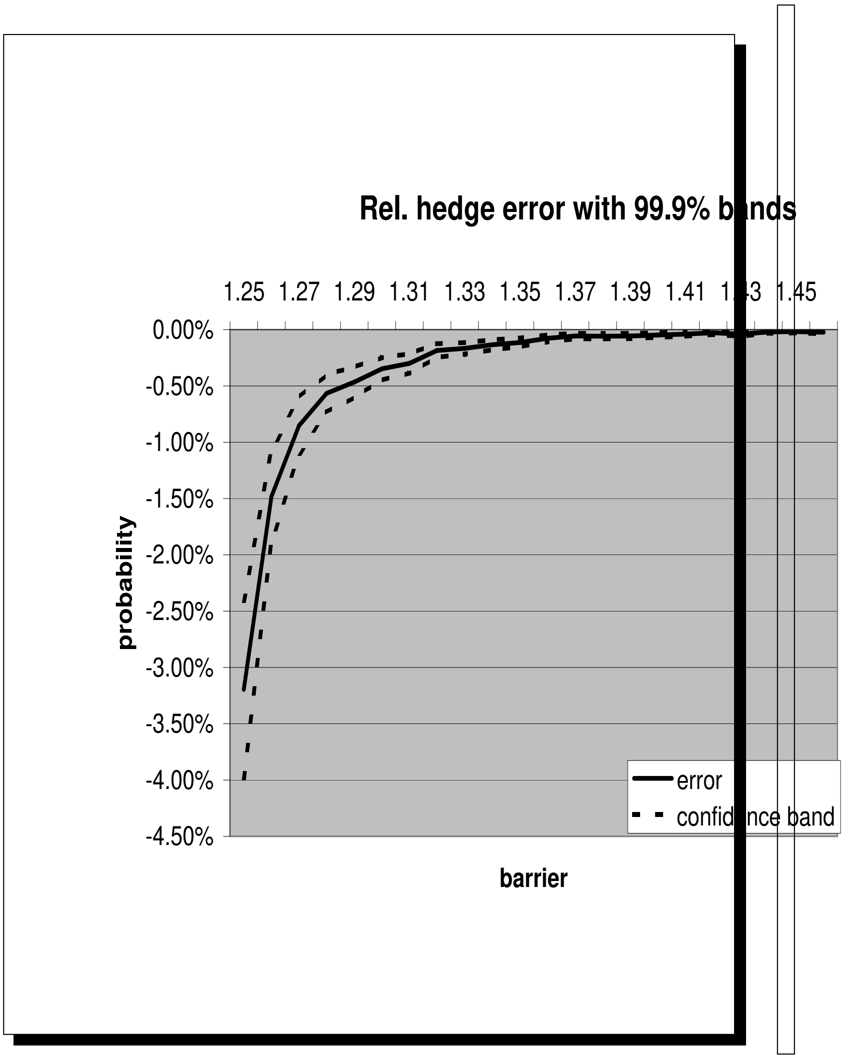


3.3 Probability of a Miss-Hedge



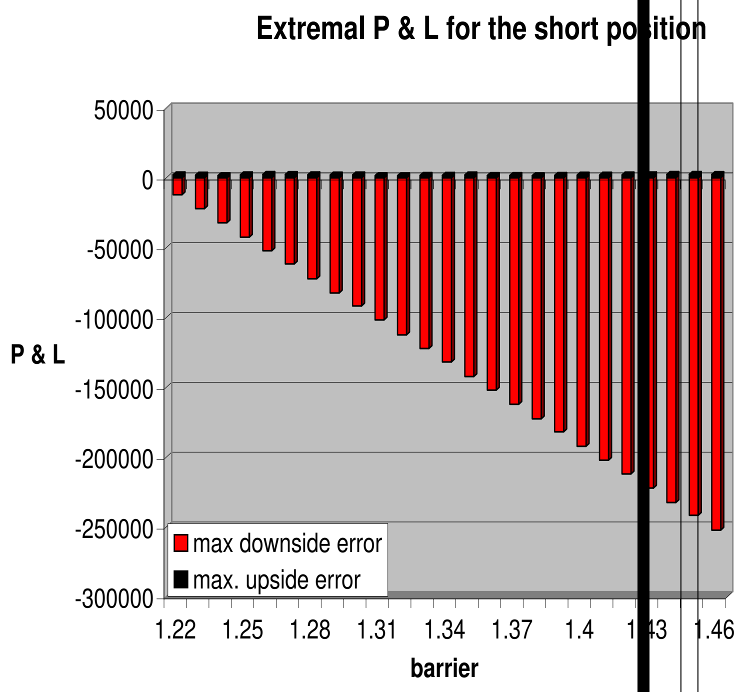
3.4 Hedging Error / TV



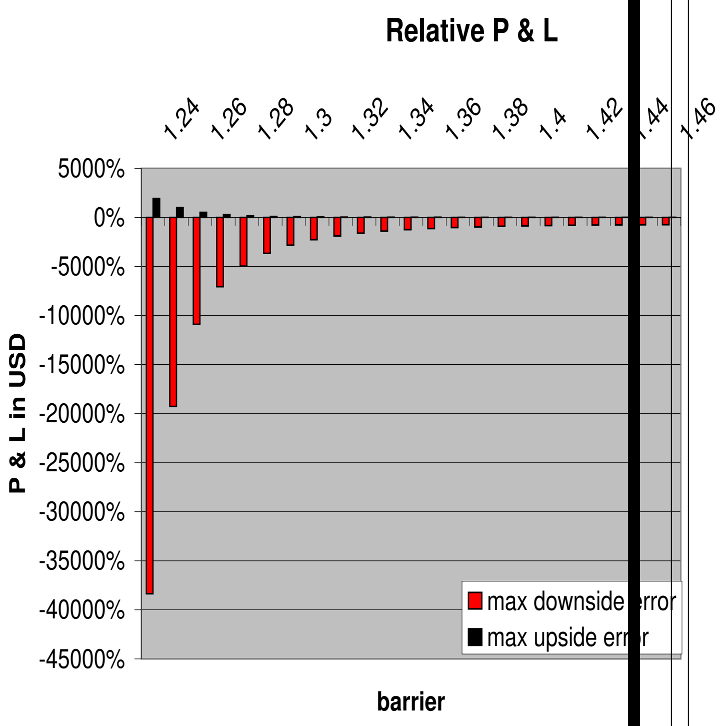


Rel. hedge error with 99.9% bands
1.25
1.27
1.29
1.31
1.33
1.35
1.37
1.39
1.41
1.45
0.00%
-0.50%
-1.00%
-1.50%
-2.00%
-2.50%
-3.00%
-3.50%
-4.00%
-4.50%
probability
barrier
error

3.5 Maximum Losses



3.6 Maximum Losses / TV



4 Summary

- other currency pairs are similar.
- average loss is comparatively small for liquid currency pairs.
- maximum loss can be very large with small probability.
- sufficient to charge a maximum of 0.1% of the TV to cover the potential *average* loss.
- traders take extra premium of 10 basis points per unit of the notional of the underlying.
- relative errors are so small that it seems reasonable not to pursue any further investigation with other models beyond Black-Scholes.

5 Contact Information

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`http://www.mathfinance.de/wystup/papers/fixingdelay.pdf`
These slides are available at
`http://www.mathfinance.de/wystup/papers/fixingdelay_slides.pdf`

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