On the Cost of Delayed Currency Fixing Announcements*

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Abstract

In Foreign Exchange Markets vanilla and barrier options are traded frequently. The market standard is a cutoff time of 10:00 a.m. in New York for the strike of vanillas and a knock-out event based on a continuously observed barrier in the inter bank market. However, many clients, particularly from Italy, prefer the cutoff and knock-out event to be based on the fixing published by the European Central Bank on the Reuters Page ECB37. These barrier options are called discretely monitored barrier options. While these options can be priced in several models by various techniques, the ECB source of the fixing causes two problems. First of all, it is not tradable, and secondly it is published with a delay of about 10 - 20 minutes. We examine here the effect of these problems on the hedge of those options and consequently suggest a cost based on the additional uncertainty encountered.

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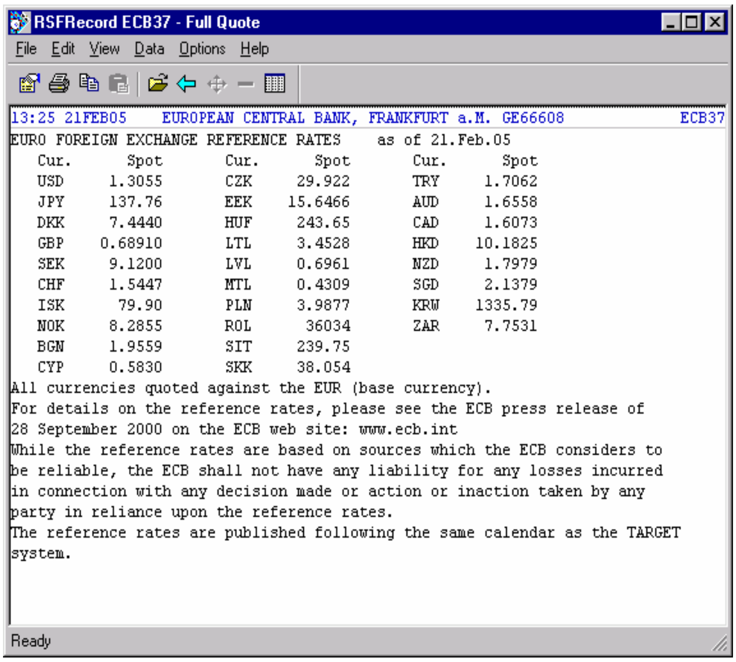
1 Introduction

1.1 The Currency Fixing of the European Central Bank

The European Central Bank (ECB) sets currency fixings every working day in Frankfurt at 2:15 p.m. Frankfurt time. The actual procedure of this fixing is done by observing the spot rates in the inter bank market, in which the ECB also participates. Traders of the ECB in various locations get together to decide on how to set the fixing. The quantity quoted is not a bid price or an offer price the ECB or anybody else necessarily trades at, but is rather used for statistical and official means, for instance tax computation or economic research. An example of the ECB37 REUTERS screen is presented in Figure 1.


RSFRecord ECB37 - Full Quote
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13:25 21FEB05 EUROPEAN CENTRAL BANK, FRANKFURT a.M. GE66608 ECB37
EURO FOREIGN EXCHANGE REFERENCE RATES as of 21.Feb.05

Cur.	Spot	Cur.	Spot	Cur.	Spot
USD	1.3055	CZK	29.922	TRY	1.7062
JPY	137.76	EEK	15.6466	AUD	1.6558
DKK	7.4440	HUF	243.65	CAD	1.6073
GBP	0.68910	LTL	3.4528	HKD	10.1825
SEK	9.1200	LVL	0.6961	NZD	1.7979
CHF	1.5447	MTL	0.4309	SGD	2.1379
ISK	79.90	PLN	3.9877	KRW	1335.79
NOK	8.2855	ROL	36034	ZAR	7.7531
BGN	1.9559	SIT	239.75		
CYP	0.5830	SKK	38.054		

All currencies quoted against the EUR (base currency).
For details on the reference rates, please see the ECB press release of 28 September 2000 on the ECB web site: www.ecb.int
While the reference rates are based on sources which the ECB considers to be reliable, the ECB shall not have any liability for any losses incurred in connection with any decision made or action or inaction taken by any party in reliance upon the reference rates.
The reference rates are published following the same calendar as the TARGET system.

Ready

Figure 1: Reuters screen ECB37 of 21 February 2005 showing the fixings of all currencies against EUR

Corporate treasures often prefer an independent source for currency exchange rates that provides a reference rate for their underlying under consideration. This way they are not bound to their own bank that might move the quoted cut-off rate in favor of their own position. The key features to stress are the following.

1. The ECB fixing is *not* tradable.

2. The ECB fixing is published with a delay of 10-20 minutes.

In this paper we analyze the impact on the value for the short position of a discretely monitored reverse knock-out, as the problems mentioned above impose additional uncertainty when it comes to determining a proper hedge. Most of the hedging error is expected in the case of jumps in the payoff of the option, which is why we restrict ourselves to the liquidly traded up-and-out call option. The currency-pairs under consideration are EUR-USD, USD-JPY, USD-CHF and EUR-GBP.

1.2 Model and Payoff

To model the exchange rate we choose a geometric Brownian motion,

$$dS_t = S_t[(r_d - r_f)\,dt + \sigma\,dW_t], \tag{1}$$

under the risk-neutral measure. As usual, r_d denotes the domestic interest rate, r_f the foreign interest rate, σ the volatility. These parameters are assumed to be constant in this paper. For contract parameters maturity in years T, strike K and knock-out barrier B, fixing schedule $0 = t_0 < t_1 < t_2 \ldots, t_n = T$, the payoffs for the vanilla and for a discretely monitored up-and-out barrier option under consideration are

$$V(F_T, T) = (\phi(F_T - K))^+, \tag{2}$$

$$V(F, T) = (\phi(F_T - K))^+ \mathbb{I}_{\{\max(F_{t_0},\ldots,F_{t_n})<B\}}, \tag{3}$$

where F_t denotes the fixing of the underlying exchange rate at time t, $\mathbb{I}$ the indicator function and ϕ a put-call indicator taking the value $+1$ for a call and -1 for a put. Of course, F_t is usually close to S_t, the spot at time t, but it may differ as well. We start with payoffs

$$V(S_T, T) = (\phi(S_T - K))^+, \tag{4}$$

$$V(S, T) = (\phi(S_T - K))^+ \mathbb{I}_{\{\max(S_{t_0},\ldots,S_{t_n})<B\}}, \tag{5}$$

whose values are explicitly known in the Black-Scholes model. In this model, the values are called *theoretical value (TV)*.

1.3 Analysis Procedure

1. We simulate the spot process with a Monte Carlo simulation using an Euler-discretization. Furthermore, we use a *Mersenne Twister* pseudo random number generator by Takuji Nishimura and Makoto Matsumoto [5] and a library to compute the inverse of the normal cumulative distribution function written by Barry W. Brown et al. [2].

2. We model the ECB-fixing F_t by

$$F_t = S_t + \varphi, \quad \varphi \in \mathcal{N}(\mu, \sigma), \tag{6}$$

where μ and σ are estimated from historic data. Note that F_t denotes the ECB-fixing at time t, which is nonetheless only announced 10 - 20 minutes later. We denote this time delay by $\Delta_{ECB}(T)$. This means that we model the error, i.e. the difference of fixing and traded spot, as a normally distributed random variable. The estimated values for the mean and the standard-deviation of the quantity Spot - ECB Fixing from historic time series are listed in Table 1. For the cross rates, where EUR is not one of the currencies, we take the respective ratios of fixings against EUR, e.g. for the USD/JPY fixing we divide the EUR/JPY fixing by the EUR/USD fixing, which is also common market practise in trade confirmations.

Currency pair	Expected value	Standard deviation	Time horizon
EUR / USD	-3.125E-6	0.0001264	23.6 - 08.8.04
USD / YEN	-4.883E-3	0.0134583	22.6 - 26.8.04
USD / CHF	-1.424E-5	0.0001677	11.5 - 26.8.04
EUR / GBP	-1.33E-5	0.00009017	04.5 - 26.8.04

Table 1: Estimated values for mean and standard-deviations of the quantity Spot - ECB-fixing from historic time series. The time series were provided by Commerzbank AG

3. We evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold. Then we compute for each path the error encountered due the fixing being different from the spot, and then average over all paths.
4. We do this for various currency pairs, parameter scenarios, varying the rates, volatilities, maturities, barriers and strikes. We expect a significant impact particularly for reverse knock-out barrier options due to the jump of the payoff and hence the large delta hedge quantity.

2 Error Estimation

Note that since we expect the resulting errors to be fairly small, we introduce a bid/offer-spread δ for the spot, which is of the size of 2 basis points in the inter bank market. We consider the following options in detail.

2.1 European Style up-and-out Call

To determine the possible hedging error we propose the following to be appropriate. Note that the error is measured for a nominal of 1 unit of the underlying. We consider three cases.

1. Let $S_T \leq K$. In this case, the seller who is short the option decides not to hedge as the option is probably out of the money, i.e. delta = 0. If the option turns out to be in the money, i.e. $F_T > K$, the holder of the short position faces a P&L of $K - (S(T + \Delta_{ECB}(T)) + \delta)$ (units of the base currency).
2. Let $S_T > K$ and $S_T < B$. Hence, one assumes that the option is in the money and delta is 1. If now $F_T \leq K$ or $F_T \geq B$, there is a P&L of $S(T + \Delta_{ECB}(T)) - (S(T) + \delta)$.

3. Let $S_T \geq B$ and $F_T < B$. Here we have a P&L of $K-(S(T+\Delta_{ECB}(T))+\delta)$. Note that other than in the first case, this P&L is of order $K-B$ due to the jump in the payoff.

2.2 Discretely Monitored up-and-out Call

We consider a time to maturity of one year with 250 knock-out-events, i.e., the possible knock out occurs every working day at 2:15 p.m. Frankfurt time, when the ECB fixes the reference rate of the underlying currency pair. We propose the following error determination to be appropriate. First of all, we adopt the procedure above for the maturity time. In addition, we consider every knock-out-event and examine the following cases.

1. Let $S_t < B$ and $F_t \geq B$. At time t the trader holds $\Delta(S_t)$ shares of stock in the delta hedge. He does not unwind the hedge at time t, as the spot is below the barrier. Only after the fixing announcement, it turns out that the hedge needs to be unwound, so he does this with delay and encounters a P&L of

$$\Delta(S_t) \cdot (S_{t+\Delta_{ECB}(T)} - S_t), \tag{7}$$

   where $\Delta(S_t)$ denotes the theoretical delta of the option under consideration, if the spot is at S_t. To see this, it is important to note, that the theoretical delta is negative if the underlying is near the barrier B. In this way, the seller of the option has been short the underlying at time t and must buy it in $t+\Delta_{ECB}(T)$ minutes to close out the hedge. Therefore, he makes profit if the underlying is cheaper in $t+\Delta_{ECB}(T)$, which is reflected in our formula. We shall elaborate later how to compute the theoretical delta, but we would like to point out that whenever we need a spot price at time t to calculate such a delta or to compute the value of a hedge, we refer to S as the tradable instrument instead of the contractually specified underlying F in order to account for the ECB fixing being non-tradable.

2. Let $S_t \geq B$ and $F_t < B$. Here the seller of the option closed out the hedge at time t, though she shouldn't have done so, and in $t+\Delta_{ECB}(T)$ she needs to build a new hedge. Note again that the theoretical delta is negative. This means that at time t the seller bought the underlying with the according theoretical delta-quantity, and in $t+\Delta_{ECB}(T)$ she goes short the underlying with the appropriate new delta-quantity. The profit and loss (P&L) is calculated via

$$\text{P\&L} = \Delta(S_t) \cdot (S_t + \delta) - \Delta(S_{t+\Delta_{ECB}(T)}) \cdot S_{t+\Delta_{ECB}(T)} \tag{8}$$

The other cases do not lead to errors due to an unexpected fixing announcement. Of course, delta hedging an option in the Black-Scholes model can lead to errors, because of hedge adjustments at discrete times and and because of model risk in general, see, e.g. [1].

2.3 Calculating the Delta-Hedge Quantity

The valuation of continuously monitored barrier options has been treated, e.g., in [3]. In order to compute the theoretical delta for the discretely monitored up-and-out call, for which no closed-form solution is known, in acceptable time and precision, we refer to an approximation proposed by Per Hörfelt in [4], which works in the following way. Assume the value of the spot is observed at times iT/n, $i = 0, \ldots, n$, and the payoff of the discretely monitored up-and-out call is given by Equation (5). We define the value and abbreviations

$$\theta_\pm \triangleq \frac{r_d - r_f \pm \sigma^2/2}{\sigma}\sqrt{T}, \tag{9}$$

$$c \triangleq \frac{\ln(K/S_0)}{\sigma\sqrt{T}}, \tag{10}$$

$$d \triangleq \frac{\ln(B/S_0)}{\sigma\sqrt{T}}, \tag{11}$$

$$\beta \triangleq -\zeta(1/2)/\sqrt{(2\pi)} \approx 0.5826, \tag{12}$$

where ζ denotes the Riemann zeta function. We define the function

$$F_+(a, b; \theta) \triangleq \mathcal{N}(a - \theta) - e^{2b\theta}\mathcal{N}(a - 2b - \theta) \tag{13}$$

and obtain for the value of the discretely monitored up-and-out call

$$\begin{aligned} V(S_0, 0) \approx\ & S_0 e^{-r_f T}\left[F_+(d, d + \beta/\sqrt{n}; \theta_+) - F_+(c, d + \beta/\sqrt{n}; \theta_+)\right] \\ & -K e^{-r_d T}\left[F_+(d, d + \beta/\sqrt{n}; \theta_-) - F_+(c, d + \beta/\sqrt{n}; \theta_-)\right]. \end{aligned} \tag{14}$$

Using this approximation for the value, we take a finite difference approach for the computation of the theoretical delta,

$$\Delta = V_S(S, t) \approx \frac{V(S + \epsilon, t) - V(S - \epsilon, t)}{2\epsilon}. \tag{15}$$

3 Analysis of EUR-USD

Considering the simulations for a maturity T of one year, huge hedging errors can obviously only occur near the barrier. The influence of the strike is comparatively small, as we discussed in the error determination procedure above. In this way we chose the values listed in Table 2 to remain constant and only to vary the barrier.

Using Monte Carlo simulations with one million paths we show the *average* of the profit and loss with 99.9% confidence bands and how the probability of a mishedge depends on the position of the barrier in Figure 2. It appears that the additional costs for the short position are negligible. We also learn that the mishedge is larger for a barrier in a typical traded distance from the spot, i.e. not too close and not too far.

In Figure 3 we plot the barrier against the ratio Hedging Error / TV of the up-and-out call and the ECB-fixing as underlying. This relationship is an important message for the risk-averse trader. For a one-year reverse knock-out call we see an average relative hedge error below 5% of TV if the barrier is at least 4 big

Spot	1.2100
Strike	1.1800
Trading days	250
Domestic interest rate	2.17% (USD)
Foreign interest rate	2.27% (EUR)
Volatility	10.4%
Time to maturity	1 year
Notional	1,000,000 EUR

Table 2: EUR-USD testing parameters

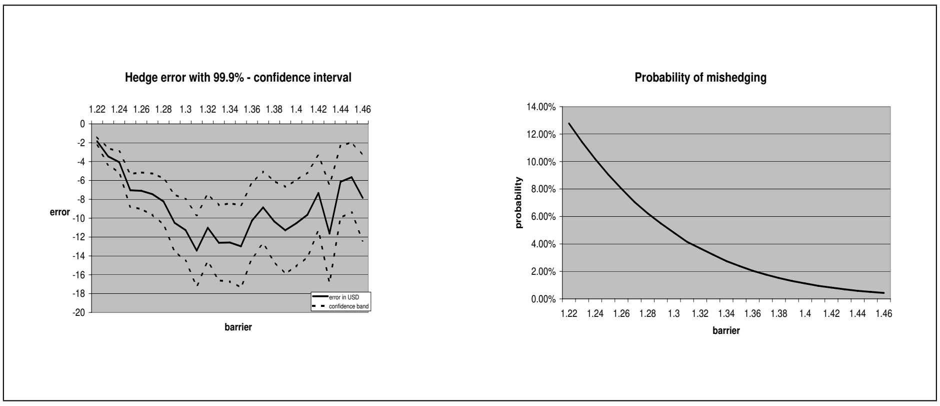

Figure 2: Additional average hedge costs and probability of a mishedge for the short position of a discretely monitored up-and-out call in EUR-USD

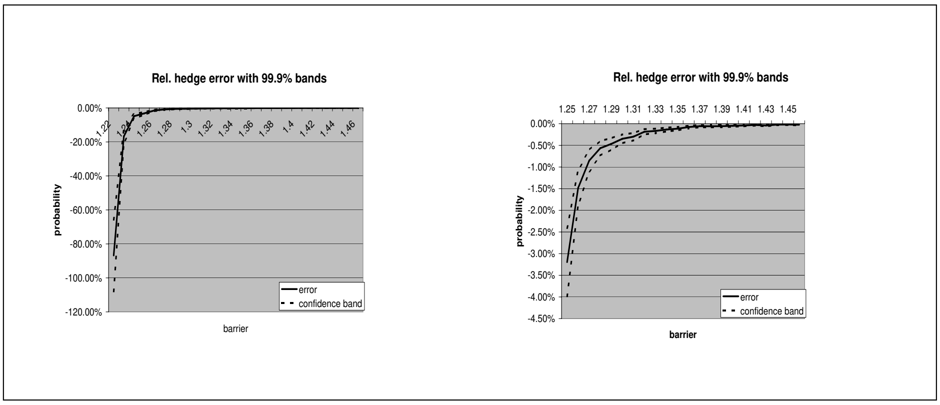

Figure 3: Hedging Error /TV for a discretely monitored up-and-out call in EUR-USD

figures away from the spot. Traders usually ask for a risk premium of 10 basis points.

Finally, we would like to point out that the *average* loss is not the full story as an average is very sensitive to outliers. Therefore, we present in Figure 4 the distribution of the maximal profits and losses, both in absolute as well as in relative numbers. The actual numbers are presented in Table 3. We have found other parameter scenarios to behave similarly. The crucial quantity is the intrinsic value at the knock-out barrier. The higher this value, the more dangerous the trade. In particular we do not exhibit the results for the vanilla as there is hardly anything to see. Varying rates and volatilities do not yield any noticeably different results.

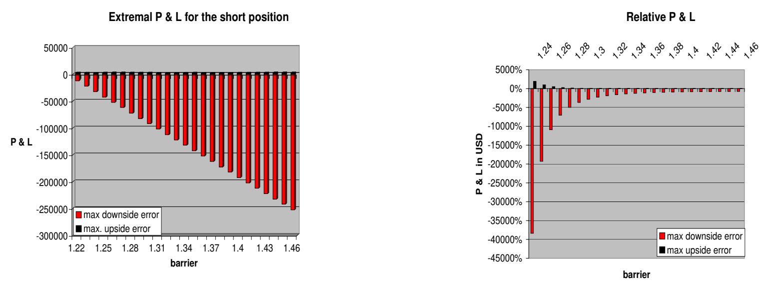

Figure 4: Absolute and relative maximum profit and loss distribution for a discretely monitored up-and-out call in EUR-USD. The upside error is the unexpected gain a trader will face. The downside error is his unexpected loss. On average the loss seems small, but the maximum loss can be extremely high. The effect is particularly dramatic for knock-out calls with a large intrinsic value at the barrier as shown in the left hand side. The right hand side shows the maximum gain and loss relative to the TV. Of course, the further the barrier is away from the spot, the smaller the chance of hedging error occurring.

barrier 1.2500	<1k$	<2k$	<3k$	<39k$	<40k$	<41k$	<42k$	<43k$
upside error	951744	20	1	0	0	0	0	0
downside error	48008	54	2	5	59	85	21	1
barrier 1.3000	<1k$	< 2k$	< 3k$	<89k$	< 90k$	<91k$	<92k$	
upside error	974340	20	1	0	0	0	0	
downside error	25475	43	0	2	40	59	20	
barrier 1.4100	<1k$	<2k$	<3k$	<199k$	<200k$	<201k$	<202k$	<203k$
upside error	994854	78	0	0	0	0	0	0
downside error	4825	194	3	1	19	17	8	1

Table 3: EUR-USD distribution of absolute errors in USD. The figures are the number of occurrences out of 1 million. For instance, for the barrier at 1.2500, there are 54 occurrences out of 1 million, where the trader faces a loss between 1000 and 2000 $.

As the analysis of the other currency pairs is of similar nature, we list it in the appendix and continue with the conclusion.

4 Conclusion

We have seen that even though a trader can be in a time interval where he does not know what delta hedge he should hold for an option due to the delay of the fixing announcement, the loss encountered is with probability 99.9% within less than 5% of the TV for usual choices of barriers and strikes and the liquid currency pairs, in which complex barrier options such as a discretely monitored up-and-out call are traded. However, the maximum loss quantity in case of a hedging error can be rather substantial. So in order to take this into account, it appears generally sufficient to charge a maximum of 10% of the TV to cover the potential loss with probability 99.9%. This work shows that the extra premium of 10 basis points per unit of the notional of the underlying, which traders argue is needed when the underlying is the ECB-fixing instead of the spot, is justified and well in line with our results. However, charging 10 basis points extra may be easy to implement, but is not really precise as we have seen, since the error depends heavily on the distance of the barrier from the spot.

Of course the level of complexity of the model can be elaborated further arbitrarily, but using a geometric Brownian motion and a Monte Carlo simulation appears sufficient. The relative errors are small enough not to pursue any further investigation concerning this problem.

5 Appendix

5.1 Analysis of USD-CHF

We used the market and contract data listed in Table 4. We summarize the results in Table 5 and Figure 5. The results are similar to the analysis of EUR-USD.

Spot	1.2800
Strike	1.2000
Trading days	250
Domestic interest rate	0.91% (CHF)
Foreign interest rate	2.17% (USD)
Volatility	11.25%
Time to maturity	1 year
Notional	1,000,000 USD

Table 4: USD-CHF testing parameters

barrier 1.3300	< 1k	< 2k	< 3k	< 49k	< 50k	< 51k	< 52k	< 54k
upside error	950902	29	1	0	0	0	0	0
downside error	48846	65	2	9	44	72	28	2
barrier 1.3900	< 1k	< 2k	< 3k	< 109k	< 110k	< 111k	< 112k	< 113k
upside error	975939	26	3	0	0	0	0	0
downside error	23847	72	5	4	23	52	27	2
barrier 1.5000	< 1k	< 2k	< 3k	< 4k	< 5k	< 219k	< 220k	< 222k
upside error	993544	749	90	2	0	0	0	0
downside error	4523	880	158	12	3	2	12	25

Table 5: USD-CHF distribution of absolute errors in CHF. The figures are the number of occurrences out of 1 million

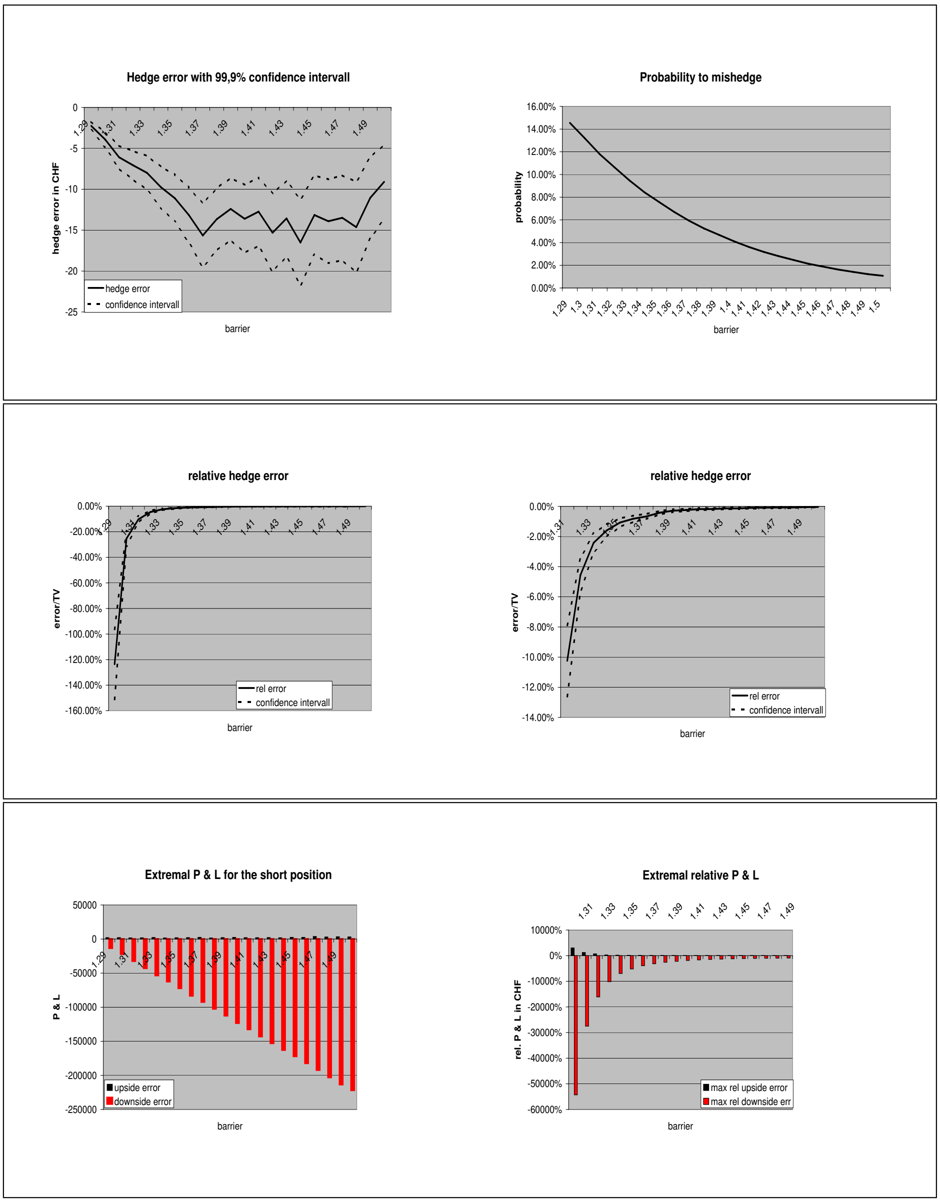

Figure 5: Analysis of the discretely monitored up-and-out call in USD-CHF

5.2 Analysis of USD-JPY

We used the market and contract data listed in Table 6. Again the analysis looks very much like the EUR-USD case. We summarize the results in Table 7 and Figure 6.

Spot	110.00
Strike	100.00
Trading days	250
Domestic interest rate	0.03% (JPY)
Foreign interest rate	2.17% (USD)
Volatility	9.15%
Time to maturity	1 year
Notional	1,000,000 USD

Table 6: USD-JPY testing parameters

barrier 133.00	< 10k	<20k	< 30k	< 40k	< 50k	< 60k	< 70k
upside error	949563	957	29	34	18	20	19
downside error	47623	1260	32	25	23	23	21
barrier 133.00	< 80k	< 155k	< 175k	more			
upside error	21	39	3	0			
downside error	13	34	0	234			

Table 7: USD-JPY distribution of absolute errors in JPY. The figures are the number of occurrences out of 1 million

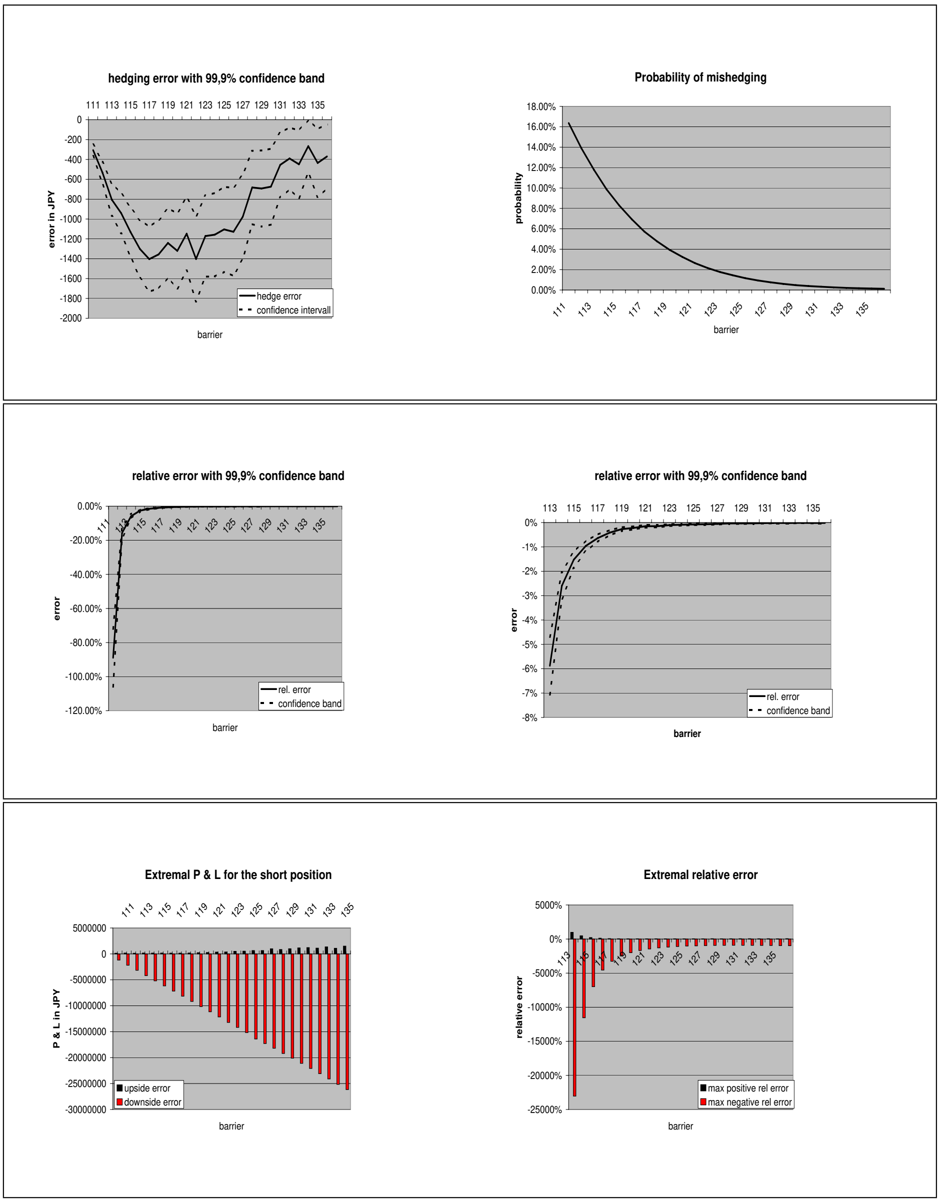

Figure 6: Analysis of the discretely monitored up-and-out call in USD-JPY

5.3 Analysis of EUR-GBP

To compare we take now EUR-GBP as a currency pair without the USD. We used the market and contract data listed in Table 8. Again, we observe the usual picture, i.e., a similar hedging error curvature as in EUR-USD. We summarize the results in Table 9 and Figure 7.

Spot	0.6700
Strike	0.6700
Trading days	250
Domestic interest rate	5.17% (GBP)
Foreign interest rate	2.27% (EUR)
Volatility	7.65%
Time to maturity	1 year
Notional	1,000,000 EUR

Table 8: EUR-GBP testing parameters

barrier 0.7100	< 1k	< 2k	< 4k0	< 41k	< 42k
upside error	950005	0	0	0	0
downside error	49711	3	59	220	2
barrier 0.7400	< 1k	< 2k	< 7k0	< 71k	< 72k
upside error	978884	139	0	0	0
downside error	20493	273	48	162	1
barrier 0.7800	< 1k	< 2k	< 3k	< 4k	< 5k
upside error	994032	678	122	11	2
downside error	4001	860	183	27	2
barrier 0.7800	< 109k	< 11k0	< 111k	< 112k	< 114k
upside error	0	0	0	0	0
downside error	1	15	63	2	1

Table 9: EUR-GBP distribution of absolute errors in GBP. The figures are the number of occurrences out of 1 million

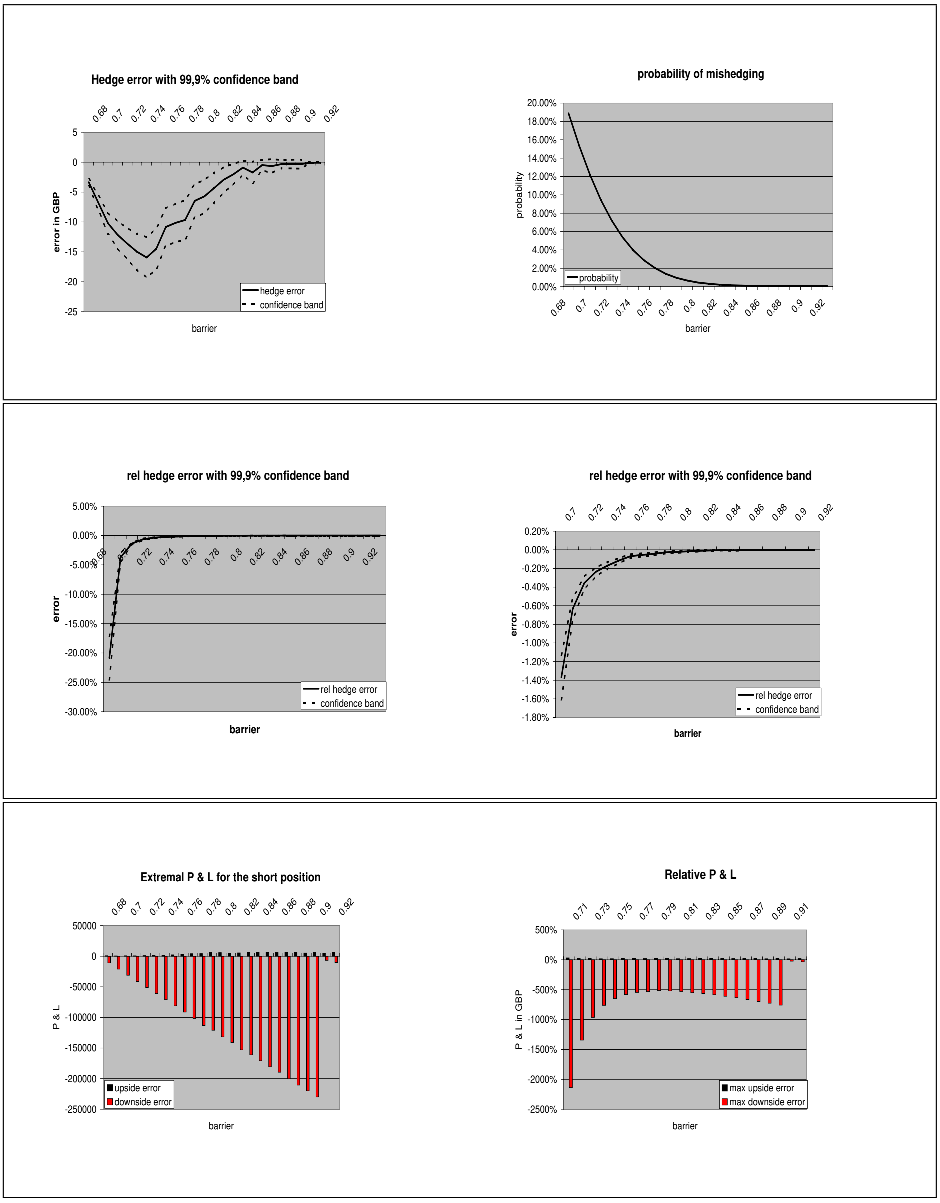

Figure 7: Analysis of the discretely monitored up-and-out call in EUR-GBP

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