Making the Best out of Multiple Currency Exposure: Protection with Basket Options

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In many cases corporate and institutional currency managers are faced with an exposure in more than one currency. Generally these exposures would be hedged using individual strategies for each currency. These strategies are composed of spot transactions, forwards, and in many cases options on a single currency. Nevertheless, there are instruments that include several currencies, and these can be used to build a multi-currency strategy that is almost always cheaper than the portfolio of the individual strategies. As a prominent example we explain basket options in detail.

1. Basket Options

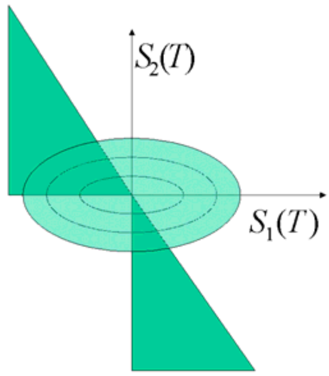
Basket options are derivatives based on a common base currency, say €, and several other risky currencies. The option is actually written on the basket of risky currencies. Basket options are European options paying the difference between the basket value and the strike, if positive, for a basket call, or the difference between strike and basket value, if positive, for a basket put respectively at maturity. The risky currencies have different weights in the basket to reflect the details of the exposure.

For example, a basket call on two currencies US-\$ and JP-¥ pays off

$$\max\left(a_1\frac{S_1(T)}{S_1(0)}+a_2\frac{S_2(T)}{S_2(0)}-K;0\right)$$

at maturity T, where $S_1(t)$ denotes the exchange rate of €/\$ and $S_2(t)$ denotes the exchange rate of €/¥ at time t, a_i the corresponding weights and K the strike.

A basket option protects against a drop in both currencies at the same time. Individual options on each currency cover some cases, which are not protected by a basket option (shaded triangular areas) and that's why they cost more than a basket.



The ellipsoids connect the points that are reached with the same probability assuming that the forward prices are at the centre.

2. Pricing Basket Options

Basket options should be priced in a consistent way with plain vanilla options. Hence the basic model assumption is a log-normal process for the individual correlated basket components. A decomposition into uncorrelated components of the exchange rate processes $dS_i=\mu_i S_i dt+S_i\sum_{j=1}^{N}\Omega_{ij}dW_j$ is the basis for pricing. Here μ_i denotes the difference between the foreign and the domestic interest rate of the i-th currency pair, dW_j the j-th component of independent Brownian increments. The covariance matrix is given by $C_{ij}=(\Omega\Omega^T)_{ij}=\rho_{ij}\sigma_i\sigma_j$. Here σ_i denotes the volatility of the i-th currency pair and ρ_{ij} the correlation coefficients.

2.1. Exact Method

Starting with the uncorrelated components the pricing problem is reduced to the N-dimensional integration of the payoff. This method is accurate but rather slow for more than two or three basket components.

2.2. A Simple Approximation

A simple approximation method assumes that the basket spot itself is a log-normal process with drift μ and volatility σ driven by a Wiener Process $W(t)$

$$dS(t) = S(t)\left[\mu dt + \sigma dW(t)\right]$$

with solution

$$S(T) = S(t) e^{\sigma W(T-t) + \left(\mu - \frac{1}{2}\sigma^2\right)(T-t)}$$

given we know the spot $S(t)$ at time t. It is a fact that the sum of log-normal processes is not log-normal itself, but as a crude approximation it is certainly a quick method that is easy to implement. In order to price the basket call the drift and the volatility of the basket spot need to be determined. This is done by matching the first and second moment of the basket spot with the first and second moment of the log-normal model for the basket spot. The moments of log-normal spot are:

$$\mathrm{E}\left(S(T)\right) = S(t) e^{\mu(T-t)}$$
$$\mathrm{E}\left(S(T)^2\right) = S(t)^2 e^{(2\mu+\sigma^2)(T-t)}$$

We solve these equations for the drift and volatility

$$\mu = \frac{1}{T-t} \ln\left(\frac{E\left(S(T)\right)}{S(t)}\right)$$
$$\sigma = \sqrt{\frac{1}{T-t} \ln\left(\frac{E\left(S(T)^2\right)}{E\left(S(T)\right)^2}\right)}$$

In these formulae we now use the moments for the basket spot:

$$\mathrm{E}\left(S(T)\right) = \sum_{i=1}^{N} \alpha_i S_i(t) e^{\mu_i (T-t)}$$
$$\mathrm{E}\left(S(T)^2\right) = \sum_{i,j=1}^{N} \alpha_i \alpha_j S_i(t) S_j(t) e^{\left(\mu_i + \mu_j + \sum_{k=1}^{N} \Omega_{ki}\Omega_{jk}\right)(T-t)}$$

The pricing formula is the well-known Black-Scholes-Merton formula for plain vanilla call options:

$$\upsilon(0) = e^{-r_d T}\left(F\boldsymbol{N}(d_+) - K\boldsymbol{N}(d_-)\right)$$
$$F = S(0) e^{\mu T}$$
$$d_\pm = \frac{1}{\sigma\sqrt{T}}\left(\ln\left(\frac{F}{K}\right) \pm \frac{1}{2}\sigma^2 T\right)$$

Here $\boldsymbol{N}$ denotes the cumulative normal distribution function and r_d the domestic interest rate.

2.3. A More Accurate and Equally Fast Approximation

The previous approach can be taken one step further by introducing one more term in the Itô-Taylor expansion of the basket spot, which results in

$$\upsilon(0) = e^{-r_d T}\left(F\mathbf{N}(d_1) - K\mathbf{N}(d_2)\right)$$

$$F = \frac{S(0)}{\sqrt{1-\lambda T}} e^{\left(\mu - \frac{\lambda}{2} + \frac{\lambda\sigma^2}{2(1-\lambda T)}\right)T}$$

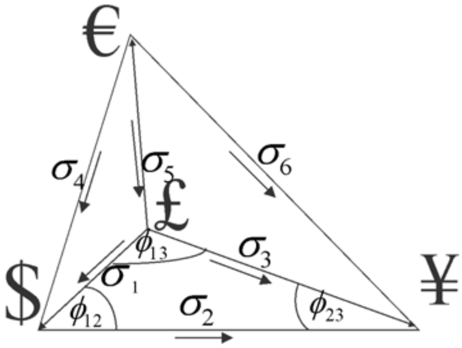
$$d_2 = \frac{\sigma - \sqrt{\sigma^2 + \lambda\left(\left(1 + \frac{\lambda}{1-\lambda T}\right)\sigma^2 T - 2\ln\frac{F\sqrt{1-\lambda T}}{K}\right)}}{\sqrt{T}\lambda}$$

$$d_1 = \sqrt{1-\lambda T}\, d_2 + \frac{\sigma\sqrt{T}}{\sqrt{1-\lambda T}}$$

The new parameter λ is determined by matching the third moment of the basket spot and the model spot. For details see [1]. Most remarkably this major improvement in the accuracy only requires a marginal additional computation effort.

3. Correlation Risk

Correlation coefficients between market instruments are usually not obtained easily. Either historical data-analysis or implied calibrations need to be done. However, in the foreign exchange market the cross instrument is traded as well, e.g., for the example above \$/¥ spot and options are traded, and the correlation can be determined from this contract. In fact, denoting the volatilities as in the tetrahedron,



we obtain formulae for the correlation coefficients in terms of known market implied volatilities:

$$\rho_{12} = \frac{\sigma_3^2 - \sigma_1^2 - \sigma_2^2}{2\sigma_1\sigma_2}$$

$$\rho_{34} = \frac{\sigma_1^2 + \sigma_6^2 - \sigma_2^2 - \sigma_5^2}{2\sigma_3\sigma_4}$$

This method also allows hedging correlation risk by trading FX implied volatility. For details see [1].

4. Practical Example

We want to find out how much one can save using a basket option. We take **€** as a base currency and consider a basket of three currencies \$, £ and ¥. For the volatilities we take

GBP/USD	8.9%
USD/JPY	10.1%
GBP/JPY	9.8%
EUR/USD	10.5%
EUR/GBP	7.5%
EUR/JPY	10.0%

FX implied volatilities for 3-month at-the-money vanilla options as of Nov 23 2001. Source: Reuters

The resulting correlation coefficients are

	GBP/USD	USD/JPY	GBP/JPY	EUR/USD	EUR/GBP	EUR/JPY
GBP/USD	100%	-47%	42%	71%	-19%	27%
USD/JPY	-47%	100%	60%	-53%	-18%	45%
GBP/JPY	42%	60%	100%	10%	-36%	71%
EUR/USD	71%	-53%	10%	100%	55%	52%
EUR/GBP	-19%	-18%	-36%	55%	100%	40%
EUR/JPY	27%	45%	71%	52%	40%	100%

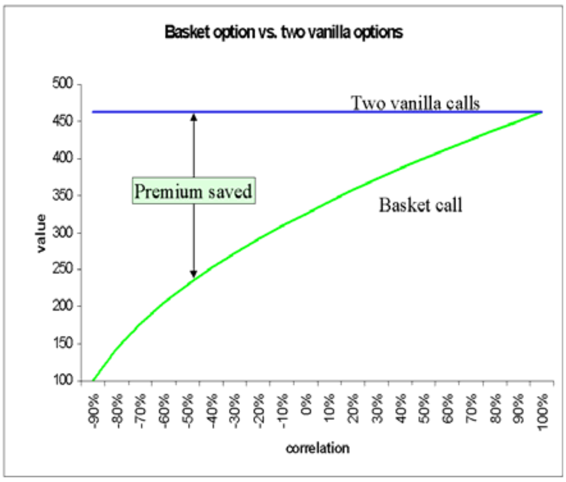
FX implied 3-month correlation coefficients as of Nov 23 2001

The amount of option premium one can save using a basket call rather than three individual call options is illustrated in the following table.

Base currency	EUR	interest rate	4.0%
nominal in EUR	39,007.00 €	strike K	1
currencies	USD	JPY	GBP
nominals	29%	30%	41%
1/spot	1.1429	0.00919	1.6091
spot	0.8750	108.81	0.6215
strikes (in EUR)	1.1432	0.00927	1.5985
volatilities	10.5%	10.0%	7.5%
interest rates	4.0%	0.5%	7.0%
BS-values (in EUR)	235	227	233
Basket value	**563**		
Sum of individuals	**695**		

Comparison of a basket call with 3 currencies for a maturity of 3-month versus the cost of 3 individual call options

The amount of premium saved essentially depends on the correlation of the currency pairs. In the next figure we take the parameters of the previous scenario, but restrict ourselves to the currencies \$ and ¥.



5. Conclusions

Many corporate clients are exposed to multi-currency risk. One way to turn this fact into an advantage is to use multi-currency hedge instruments. We have shown that basket options are convenient instruments protecting against exchange rates of most of the basket components changing in the *same* direction. A rather unlikely market move of half of the currencies' exchange rates in *opposite* directions is not protected by basket options, but when taking this residual risk into account the hedging cost is reduced substantially.

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References

[1] Hakala, J. and Wystup, U. (2001). Foreign Exchange Risk. Risk Publications, London.