MATHFINANCE

FX Column: Before Takeoff - Model Validation Checklist

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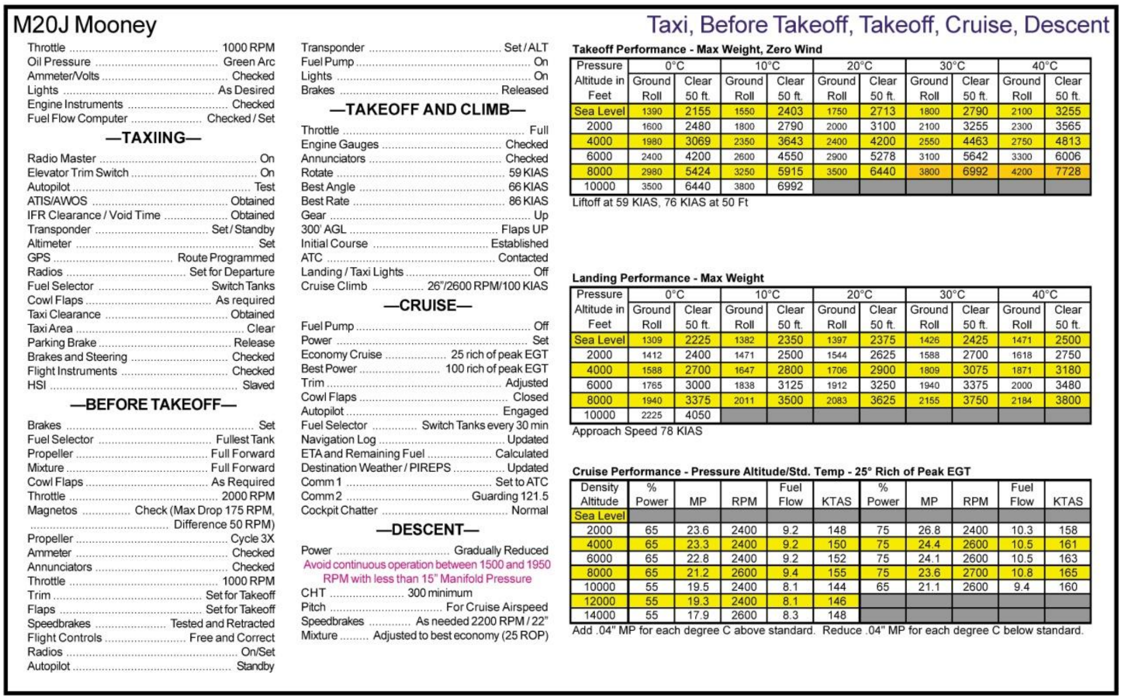
Before a pricing and risk management model can fly, it needs to be validated. Similarly, a pilot is required to check an airplane before takeoff following a checklist, so a flight is a safe and smooth as possible. There might have been the best engineers to build the engine and the fuselage, the most modern avionics, but still a checklist is required before each flight. Figure 1 illustrates the items to check to fly a Mooney 20J.



M20J Mooney

Throttle .... 1000 RPM
Oil Pressure .... Green Arc
Ammeter/Volts .... Checked
Lights .... As Desired
Engine Instruments .... Checked
Fuel Flow Computer .... Checked / Set

—TAXIING—

Radio Master .... On
Elevator Trim Switch .... On
Autopilot .... Test
ATIS/AWOS .... Obtained
IFR Clearance / Void Time .... Obtained
Transponder .... Set / Standby
Altimeter .... Set
GPS .... Route Programmed
Radios .... Set for Departure
Fuel Selector .... Switch Tanks
Cowl Flaps .... As required
Taxi Clearance .... Obtained
Taxi Area .... Clear
Parking Brake .... Release
Brakes and Steering .... Checked
Flight Instruments .... Checked
HSI .... Slaved

—BEFORE TAKEOFF—

Brakes .... Set
Fuel Selector .... Fullest Tank
Propeller .... Full Forward
Mixture .... Full Forward
Cowl Flaps .... As Required
Throttle .... 2000 RPM
Magnetos .... Check (Max Drop 175 RPM, ... Difference 50 RPM)
Propeller .... Cycle 3X
Ammeter .... Checked
Annunciators .... Checked
Throttle .... 1000 RPM
Trim .... Set for Takeoff
Flaps .... Set for Takeoff
Speedbrakes .... Tested and Retracted
Flight Controls .... Free and Correct
Radios .... On/Set
Autopilot .... Standby

Transponder .... Set / ALT
Fuel Pump .... On
Lights .... On
Brakes .... Released

—TAKEOFF AND CLIMB—

Throttle .... Full
Engine Gauges .... Checked
Annunciators .... Checked
Rotate .... 59 KIAS
Best Angle .... 66 KIAS
Best Rate .... 86 KIAS
Gear .... Up
300' AGL .... Flaps UP
Initial Course .... Established
ATC .... Contacted
Landing / Taxi Lights .... Off
Cruise Climb .... 26"/2600 RPM/100 KIAS

—CRUISE—

Fuel Pump .... Off
Power .... Set
Economy Cruise .... 25 rich of peak EGT
Best Power .... 100 rich of peak EGT
Trim .... Adjusted
Cowl Flaps .... Closed
Autopilot .... Engaged
Fuel Selector .... Switch Tanks every 30 min
Navigation Log .... Updated
ETA and Remaining Fuel .... Calculated
Destination Weather / PIREPS .... Updated
Comm 1 .... Set to ATC
Comm 2 .... Guarding 121.5
Cockpit Chatter .... Normal

—DESCENT—

Power .... Gradually Reduced
Avoid continuous operation between 1500 and 1950 RPM with less than 15" Manifold Pressure
CHT .... 300 minimum
Pitch .... For Cruise Airspeed
Speedbrakes .... As needed 2200 RPM / 22"
Mixture .... Adjusted to best economy (25 ROP)

Taxi, Before Takeoff, Takeoff, Cruise, Descent

Takeoff Performance - Max Weight, Zero Wind

Pressure Altitude in Feet	0°C		10°C		20°C		30°C		40°C	
	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.
Sea Level	1390	2155	1550	2403	1750	2713	1800	2790	2100	3255
2000	1600	2480	1800	2790	2000	3100	2100	3255	2300	3565
4000	1980	3069	2350	3643	2400	4200	2550	4463	2750	4813
6000	2400	4200	2600	4550	2900	5278	3100	5642	3300	6006
8000	2980	5424	3250	5915	3500	6440	3800	6992	4200	7728
10000	3500	6440	3800	6992						

Liftoff at 59 KIAS, 76 KIAS at 50 Ft

Landing Performance - Max Weight

Pressure Altitude in Feet	0°C		10°C		20°C		30°C		40°C	
	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.	Ground Roll	Clear 50 ft.
Sea Level	1309	2225	1382	2350	1397	2375	1426	2425	1471	2500
2000	1412	2400	1471	2500	1544	2625	1588	2700	1618	2750
4000	1588	2700	1647	2800	1706	2900	1809	3075	1871	3180
6000	1765	3000	1838	3125	1912	3250	1940	3375	2000	3480
8000	1940	3375	2011	3500	2083	3625	2155	3750	2184	3800
10000	2225	4050								

Approach Speed 78 KIAS

Cruise Performance - Pressure Altitude/Std. Temp - 25° Rich of Peak EGT

Density Altitude	% Power	MP	RPM	Fuel Flow	KTAS	% Power	MP	RPM	Fuel Flow	KTAS
Sea Level										
2000	65	23.6	2400	9.2	148	75	26.8	2400	10.3	158
4000	65	23.3	2400	9.2	150	75	24.4	2600	10.5	161
6000	65	22.8	2400	9.2	152	75	24.1	2600	10.5	163
8000	65	21.2	2600	9.4	155	75	23.6	2700	10.8	165
10000	55	19.5	2400	8.1	144	65	21.1	2600	9.4	160
12000	55	19.3	2400	8.1	146					
14000	55	17.9	2600	8.3	148					

Add .04" MP for each degree C above standard. Reduce .04" MP for each degree C below standard.

Figure 1: Checklist to Fly a Mooney M20J, prepared by MetzAir

Unless all items have been checked and verified, the plane won't take off. Similarly, for an FX derivatives pricing model, even one that has been built by the best financial engineers with most current numerical methods and most modern IT, the model validation team needs to check the model and approve it before it can fly, i.e. be taken to production. And even when it is in production, it must be constantly monitored, just like a plane in cruise mode.

So here is a proposal for a model validation checklist the MathFinance team has drafted.

1. Theoretical review

- ✓ Describe model in mathematical terms
- ✓ Describe calibration mechanisms
- ✓ Describe pricing implementation
- ✓ Describe fallback mechanisms if any

2. Model suitability

- ✓ Check if the model is fit for purpose.
- ✓ Benchmark with existing market prices, or other models.

3. Model implementation correctness

- ✓ Check if the implementation is in line with the mathematical description.
- ✓ As far as possible, test various building blocks individually to demonstrate their correctness.
- ✓ For limit-cases, compare the model to existing analytic formulas. For example, in the absence of smile, the model must recover existing analytic prices for vanilla options and/or 1st generation exotics.

4. Model input requirement

- ✓ The requirements on input data must be clearly stated.
- ✓ Define allowable range on model inputs.
- ✓ Test model behavior if requirements on model inputs and ranges are breached.

5. PV and Greeks accuracy tests

- ✓ Compare PV (Present Value), Delta and Vega of vanilla options produced by the model, to those produced using Black-Scholes analytic model.
- ✓ Tests must span a wide range of currency pairs (e.g. 10 with different volatility surface patterns), a wide range of dates (e.g. weekly data over a 2-year period), and a wide range of Delta (5-delta Put to 5-Delta call).
- ✓ Tests must include at least one period of stressed market (2008 crisis, referenda, etc.)
- ✓ Criteria: PV must be within bid/offer spread, typically a couple of basis points.
- ✓ Delta, within 2% CCY1 (absolute). Vega, within 0.05% CCY1 (absolute).

6. Stress tests

- ✓ Apply artificially large bumps to volatility inputs, from -99% to +400%
  - Uniformly on ATM, RRs and BFs
  - Scale ATM only, keeping original RRs and BFs (works only for up bumps)
  - Scale RRs and BFs only, keep original ATM
  - Parallel bumps of rate curves of +/- 10% absolute
- ✓ Compare PV, Delta and Vega of vanilla options produced by the model, to those produced using Black-Scholes analytic model.

7. Convergence tests

- ✓ Model must converge when increasing numerical accuracy (increase Monte Carlo paths, increase PDE grids resolution, increase number of points in a numerical integration)

8. Value-At-Risk tests (VaR)

- ✓ Test the suitability of the model for producing VaR
- ✓ Calibration must not fail. Or if it does, a fallback mechanism must kick-in.

9. Test Fall-Back mechanism impact on Greeks

- ✓ If a fallback mechanism exists, test its impact on PV and Greek ladders. When such mechanism kicks in, it introduces non-linearities, or discontinuities in the pricing, possibly spikes in the sensitivities.

10. Identify model known caveats and limitations

- ✓ State what risk factors are ignored
- ✓ State under what circumstances the model breaks down

Generally, one should add some explanations, tables and graphs to illustrate.

Example

As an example I consider model suitability and check a couple of models to price a 1-year one-touch in EUR-JPY paying EUR as of September 8 2020 and spot reference 124.55. I assume the market data on that day was as in Table 1 (with volatility quotes and EUR rates in %, forward points in JPY per EUR).

Tenor	ATM	25RR	25BF	10RR	10BF	Forward Points	EUR Rates
1W	8.25	-0.69	0.23	-1.27	0.65	0.90	-0.6725
2W	7.86	-0.71	0.24	-1.32	0.67	1.80	-0.6777
1M	7.93	-0.87	0.24	-1.60	0.70	3.30	-0.7625
2M	8.60	-1.30	0.29	-2.41	0.91	6.90	-0.7494
3M	8.57	-1.44	0.33	-2.66	1.06	10.20	-0.7311
6M	8.33	-1.71	0.44	-3.17	1.48	15.35	-0.8260
9M	8.39	-1.94	0.47	-3.61	1.63	23.10	-0.8031
1Y	8.46	-2.10	0.49	-3.90	1.74	30.95	-0.8022
2Y	8.56	-2.43	0.50	-4.51	1.78	52.45	-0.8311

Table 1: Possible EUR-JPY market data of 8 September 2020

The graph in Figure 2 shows the price differences between model price and the Black-Scholes price of One Touch prices (TV). The models are Mixed Local Volatility (MLV)[1] with 100%, 60%, 0% mixing factor (same mixing factor for all tenors), "Mlv_Calib" and "Slv_Calib" in a Stochastic Local Volatility (SLV)[2] model are calibrated to the

[1] Uwe Wystup, Mixed Local Volatility Boosts Distribution of Exotics, FX column in Wilmott Vol 2020, Issue 110, p. 34-37, https://doi.org/10.1002/wilm.10885

[2] Uwe Wystup, Reverse Knockout Pricing Case Study: Stochastic Local Volatility versus Vanna Volga FX column in Wilmott Vol 2019, Issue 103, p. 16-17, https://doi.org/10.1002/wilm.10787

calibration instruments, with mixing factors 57% and 53% respectively. Prices of calibration instruments are indicated by the black dots. The Mustache[3] graph contains both touch contracts with lower barriers left to the spot reference of 124.55 and touch contracts with upper barriers right to the spot reference. Bid-offer in EUR-JPY one-touch contracts is about 2%, corresponding to one unit on the y-axis.

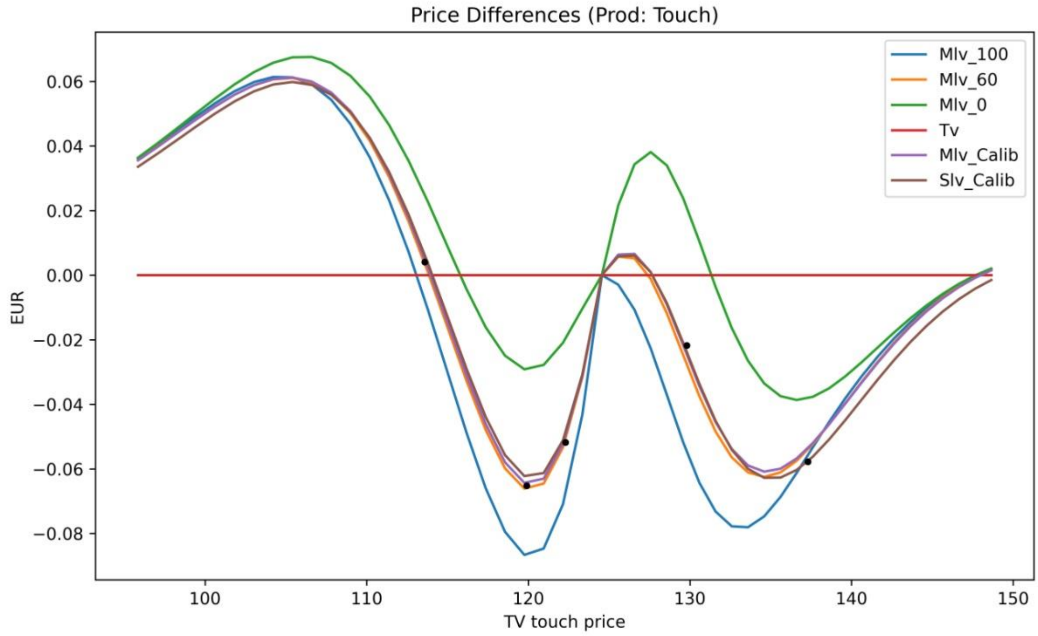


Figure 2: Price Differences for a EUR paying 1Y-OneTouch in EUR-JPY, on 8 Sept 2020

Prices of calibration instruments are indicated by the black dots. The graphs illustrate that both an MLV and SLV model with appropriately calibrated mixing factors reproduce market prices of a EUR-paying EUR-JPY One-Touch contract. In the example we have 5 touch contracts as calibration instruments. Due to the concept of calibrated MLV and SLV models all vanilla options prices are met correctly.

To see which model fits the purpose, one can conclude that the MLV model reproduces market prices equally well as an SLV model and hence one might opt for the MLV model as the calculation speed is by a factor 10 faster than in the SLV model. A comparison with other models has also been illustrated. However, this is only item 2 on the model validation checklist.

Summary

All pricing and risk management models should be thoroughly validated before putting them in production. In the example above the MLV model seem to pass parts of the checklist. However, a vanna volga model with similar mustache graphs may also appear to be suitable, however, it would typically not pass item 5, as the Greeks in a vanna-volga approach may lead to inconsistencies[4]. I would like to stress though that no model is ever perfect, but we need to find the one that passes most of the items on the checklist, and only if it does, a desk will decided whether to use a model in production. Frequent, ongoing tests must be performed, even on the way.

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[3] Uwe Wystup, Mustache to Touch, FX column in Wilmott Vol 2019, Issue 102, p. 10-11, https://doi.org/10.1002/wilm.10771
[4] Uwe Wystup, Vanna-Volga and the Greeks, FX column in Wilmott, Volume 2020, Issue 108, p. 14-16, https://doi.org/10.1002/wilm.10852

Figure 3: A Mooney 20J in Beaune between Sun and a Thunderstorm

And at the end of the day, whether or not to fly, must be decided by the pilot. The sun might be deceiving, see Figure 3. But possible thunderstorms can be detected, indeed, sometimes easily by checking the environment. Happy Landings!