The Impact of FX Options on the Spot Market and the Cost of Delayed Currency Fixing Announcements

Uwe Wystup
MathFinance AG

mailto:uwe.wystup@mathfinance.com

March 15, 2006
Abstract

In Foreign Exchange Markets vanilla and barrier options are traded frequently. In this presentation we discuss how large barrier contracts traded in the OTC market can influence and explain some of the jump-like behavior in the spot market. We furthermore show how clients insisting on official spot fixing sources give traders of barrier options a tough time hedging these options. The market standard is a cutoff time of 10:00 a.m. in New York for the strike of vanillas and a knock-out event based on a continuously observed barrier in the inter bank market. However, many clients, particularly from Italy, prefer the cutoff and knock-out event to be based on the fixing published by the European Central Bank on the Reuters Page ECB37. These barrier options are called discretely monitored barrier options. While these options can be priced in several models by various techniques, the ECB source of the fixing causes two problems. First of all, it is not tradable, and secondly it is published with a delay of about 10 - 20 minutes. We examine here the effect of these problems on the hedge of those options and consequently suggest a cost based on the additional uncertainty encountered.
1. Overview

1.1. Agenda

1. Hedging the reverse knock-out (RKO)
1. Overview

1.1. Agenda

1. Hedging the reverse knock-out (RKO)
2. Impact on the spot market
1. Overview

1.1. Agenda

1. Hedging the reverse knock-out (RKO)
2. Impact on the spot market
3. Cutoffs and fixings
1. Overview

1.1. Agenda

1. Hedging the reverse knock-out (RKO)
2. Impact on the spot market
3. Cutoffs and fixings
4. Delta-hedging a short position of a discretely monitored RKO
1. Overview

1.1. Agenda

1. Hedging the reverse knock-out (RKO)
2. Impact on the spot market
3. Cutoffs and fixings
4. Delta-hedging a short position of a discretely monitored RKO
5. Analyze cost due to unknown spot value at knock-out or expiration
2. Hedging the Reverse Knock-out (RKO)

2.1. Option Payoff

\[ F(S_T) \equiv \max(0, S_T - K) \]
2. Hedging the Reverse Knock-out (RKO)

2.1. Option Payoff

\[ F(S_T) \overset{\text{e.g.}}{=} \max(0, S_T - K) \]
2.2. Option Value

The value is

\[ v(S_t, t) = e^{-r_d T} \mathbb{E}[F(S_T)] = e^{-r_d T} \mathbb{E}[\max(0, S_T - K)] \]

\[ \ldots \text{ Black-Scholes formula} \]
2.2. Option Value

The value is

$$v(S_t, t) = e^{-rdT} \mathbb{E}[F(S_T)] \quad \text{e.g.} \quad e^{-rdT} \mathbb{E}[\max(0, S_T - K)]$$

= ... Black-Scholes formula
2.3. Delta Hedging the Short Option

The delta is the slope of the value function

$$\frac{\partial}{\partial S_t} v(S_t, t) = \ldots \text{ Black-Scholes delta}$$
2.3. Delta Hedging the Short Option

The delta is the slope of the value function

$$\frac{\partial}{\partial S_t} v(S_t, t) = \ldots \text{ Black-Scholes delta}$$
\[ \frac{\partial}{\partial S_t} v(S_t, t) = \text{delta} \]
\[
\frac{\partial}{\partial S_t} v(S_t, t) = \text{delta}
\]

- hedge a short option with value \( v(S_t, t) \)
\[
\frac{\partial}{\partial S_t} v(S_t, t) = \text{delta}
\]

- hedge a short option with value \(v(S_t, t)\)
- by buying \(\frac{\partial}{\partial S_t} v(S_t, t)\) of the underlying
\[ \frac{\partial}{\partial S_t} v(S_t, t) = \text{delta} \]

- hedge a short option with value \( v(S_t, t) \)
- by buying \( \frac{\partial}{\partial S_t} v(S_t, t) \) of the underlying
- for a short EUR-call USD put buy delta EUR
\[
\frac{\partial}{\partial S_t} v(S_t, t) = \text{delta}
\]

- hedge a short option with value \(v(S_t, t)\)
- by buying \(\frac{\partial}{\partial S_t} v(S_t, t)\) of the underlying
- for a short EUR-call USD put buy delta EUR
- ... a quantity between 0 and 100\%
2.4. Reverse Knock-Out (RKO) Call

is an option with a barrier in the money paying off

$$\max(0, S_T - K) \mathbb{I}_{\max_{0 \leq u \leq T} S_u < B}$$
2.4. Reverse Knock-Out (RKO) Call

is an option with a barrier in the money paying off

$$\max(0, S_T - K) I_{\{\max_{0 \leq u \leq T} S_u < B\}}$$
2.5. Reverse Knock-Out Call Value

\[ v(S_t, t) = e^{-r_d T} \mathbb{E} \left[ \max(0, S_T - K) \mathbb{I}_{\max_{0\leq u \leq T} S_u < B} \right] = \ldots \text{some formula} \]
2.5. Reverse Knock-Out Call Value

\[ v(S_t, t) = e^{-r_d T} \mathbb{E} \left[ \max(0, S_T - K) I_{\max_0 \leq u \leq T S_u < B} \right] \]

= \ldots \text{some formula}
2.6. Reverse Knock-Out Call Delta

Hedge a short position by buying

$$\text{delta} = \frac{\partial}{\partial S_t} v(S_t, t)$$

of the underlying.
2.6. Reverse Knock-Out Call Delta

Hedge a short position by buying

\[ \text{delta} = \frac{\partial}{\partial S_t} v(S_t, t) \]

of the underlying.
Accross time this looks like
Across time this looks like

Delta approaches $-\infty$ near maturity for a spot near the barrier.
3. Impact on the Spot Market

- by valuation theory assumptions: none
3. Impact on the Spot Market

- by valuation theory assumptions: none
- for vanilla options: none
3. Impact on the Spot Market

- by valuation theory assumptions: none
- for vanilla options: none
- for the RKO: delta = \(-1000\%\) or less is possible
3. Impact on the Spot Market

- by valuation theory assumptions: none
- for vanilla options: none
- for the RKO: $\Delta = -1000\%$ or less is possible
- implications?
3.1. How Large Barrier Contracts Affect the Market – Example

EUR/USD RKO call strike 1.2000 and barrier 1.3000
3.1. How Large Barrier Contracts Affect the Market – Example

EUR/USD RKO call strike 1.2000 and barrier 1.3000
In words:

- EUR/USD RKO call strike 1.2000 and barrier 1.3000.
In words:

- EUR/USD RKO call strike 1.2000 and barrier 1.3000.

- An investment bank delta-hedging a short position with nominal 10 Million EUR has to buy 10 Million times delta EUR. As the spot goes up to the barrier, delta becomes smaller requiring the hedging institution to sell more and more EUR.
In words:

- EUR/USD RKO call strike 1.2000 and barrier 1.3000.

- An investment bank delta-hedging a short position with nominal 10 Million EUR has to buy 10 Million times delta EUR. As the spot goes up to the barrier, delta becomes smaller requiring the hedging institution to sell more and more EUR.

- This can influence the market since steadily offering EUR slows down the spot movement towards the barrier and can in extreme cases prevent the spot from crossing the barrier.
In words:

- EUR/USD RKO call strike 1.2000 and barrier 1.3000.

- An investment bank delta-hedging a short position with nominal 10 Million EUR has to buy 10 Million times delta EUR. As the spot goes up to the barrier, delta becomes smaller requiring the hedging institution to sell more and more EUR.

- This can influence the market since steadily offering EUR slows down the spot movement towards the barrier and can in extreme cases prevent the spot from crossing the barrier.

- Once the barrier is breached, the bank has to unwind the delta hedge, buy lots of EUR ⇒ rate goes up fast
3.2. Historic Examples

Barrier options crisis in the 90s.

USD/DEM 1990-1997

low 1.3870 Sept 2 1992 1pm

all time low 1.3500 April 19 1995 9:30am

March 29 1995
10:30am 1.3800

source: Bundesbank
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
- FX: OTC standard is NY cut: Traded FX spot at 10:00 a.m. NY time
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
- FX: OTC standard is NY cut: Traded FX spot at 10:00 a.m. NY time
- Problem: not official, not transparent to public
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
- FX: OTC standard is NY cut: Traded FX spot at 10:00 a.m. NY time
- Problem: not official, not transparent to public
- Advantage: tradable, transparent to FX traders
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
- FX: OTC standard is NY cut: Traded FX spot at 10:00 a.m. NY time
- Problem: not official, not transparent to public
- Advantage: tradable, transparent to FX traders
- Other sources: FED, Warshaw Cut, Tokio Cut, Bank’s own fixing
4. Cut-Off Time and Fixings

4.1. Cut-Off

The exact time $T$ in $S_T$

- Value to take at a pre-specified time at expiration date of an option
- Source of this value must be pre-specified
- FX: OTC standard is NY cut: Traded FX spot at 10:00 a.m. NY time
- Problem: not official, not transparent to public
- Advantage: tradable, transparent to FX traders
- Other sources: FED, Warshaw Cut, Tokio Cut, Bank’s own fixing
- Average of several banks
- Example: $\text{OPTREF} = \text{AVG} (\text{COMBA, DB, DREBA, HVB})$
4.2. ECB Currency Fixing

1. Many Corporate Treasurers prefer official source of exchange rate
2. Set each business day at 2:15 p.m.
3. Published on Reuters page ECB37
4.2. ECB Currency Fixing

1. Many Corporate Treasurers prefer official source of exchange rate
2. Set each business day at 2:15 p.m.
3. Published on Reuters page ECB37
4. Not tradable
4.2. ECB Currency Fixing

1. Many Corporate Treasurers prefer official source of exchange rate
2. Set each business day at 2:15 p.m.
3. Published on Reuters page ECB37
4. Not tradable
5. Published with Delay of $\Delta T = 10-20$ Minutes
### EU ROPEXCHANGE REFERENCE RATES as of 21 Feb 2005

<table>
<thead>
<tr>
<th>Currency</th>
<th>Spot</th>
<th>EUR</th>
<th>Currency</th>
<th>Spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>1.3055</td>
<td></td>
<td>CZE</td>
<td>29.922</td>
</tr>
<tr>
<td>JPY</td>
<td>137.76</td>
<td></td>
<td>EEK</td>
<td>15.6466</td>
</tr>
<tr>
<td>DKK</td>
<td>7.4440</td>
<td></td>
<td>HUF</td>
<td>243.65</td>
</tr>
<tr>
<td>GBP</td>
<td>0.68910</td>
<td></td>
<td>LTL</td>
<td>3.4528</td>
</tr>
<tr>
<td>SEK</td>
<td>9.1200</td>
<td></td>
<td>LVL</td>
<td>0.6961</td>
</tr>
<tr>
<td>CHF</td>
<td>1.5447</td>
<td></td>
<td>MTL</td>
<td>0.4309</td>
</tr>
<tr>
<td>ISK</td>
<td>79.90</td>
<td></td>
<td>PLN</td>
<td>3.9877</td>
</tr>
<tr>
<td>NOK</td>
<td>8.2855</td>
<td></td>
<td>ROL</td>
<td>36034</td>
</tr>
<tr>
<td>BGN</td>
<td>1.9559</td>
<td></td>
<td>RON</td>
<td>38.054</td>
</tr>
</tbody>
</table>

All currencies quoted against the EUR (base currency).
For details on the reference rates, please see the ECB press release of 28 September 2000 on the ECB website: www.ecb.int
While the reference rates are based on sources which the ECB considers to be reliable, the ECB shall not have any liability for any losses incurred in connection with any decision made or action or inaction taken by any party in reliance upon the reference rates.
The reference rates are published following the same calendar as the TARGET system.
5. Delta-Hedging a Short Discretely Monitored RKO

5.1. Model

Risk-neutral geometric Brownian motion

\[ dS_t = S_t \left[ (r_d - r_f) \, dt + \sigma \, dW_t \right] \]
5. Delta-Hedging a Short Discretely Monitored RKO

5.1. Model

Risk-neutral geometric Brownian motion

\[ dS_t = S_t[(r_d - r_f) \, dt + \sigma \, dW_t] \]

These parameters are constant

- \( r_d \): domestic interest rate
- \( r_f \): foreign interest rate
- \( \sigma \): volatility
- \( S_t \): FX spot rate at time \( t \)
5.2. Contracts

- \( T \): maturity in years
- \( K \): strike
- \( B \): knock-out barrier
5.2. Contracts

- $T$: maturity in years
- $K$: strike
- $B$: knock-out barrier
- fixing schedule $0 = t_0 < t_1 < t_2 \ldots , t_n = T$
5.2. Contracts

- \( T \): maturity in years
- \( K \): strike
- \( B \): knock-out barrier
- fixing schedule \( 0 = t_0 < t_1 < t_2 \ldots, t_n = T \)
- payoff for a discretely monitored RKO call option

\[
V(F,T) = (F_T - K)^+ \mathbb{I}_{\max(F_{t_0},\ldots,F_{t_n})<B}
\]

- \( F_t \): fixing of the underlying exchange rate at time \( t \)
5.2. Contracts

- $T$: maturity in years
- $K$: strike
- $B$: knock-out barrier
- fixing schedule $0 = t_0 < t_1 < t_2 \ldots, t_n = T$
- payoff for a discretely monitored RKO call option

$$V(F, T) = (F_T - K)^+ \mathbb{I}_{\{\max(F_{t_0}, \ldots, F_{t_n}) < B\}}$$

- $F_t$: fixing of the underlying exchange rate at time $t$
- $\mathbb{I}$: the indicator function
5.2. Contracts

- $T$: maturity in years
- $K$: strike
- $B$: knock-out barrier
- fixing schedule $0 = t_0 < t_1 < t_2 \ldots, t_n = T$
- payoff for a discretely monitored RKO call option

\[
V(F, T) = (F_T - K)^+ \mathbb{I}_{\{\max(F_{t_0}, \ldots, F_{t_n}) < B\}}
\]

- $F_t$: fixing of the underlying exchange rate at time $t$
- $\mathbb{I}$: the indicator function
- payoff to hedge is

\[
V(S, T) = (S_T - K)^+ \mathbb{I}_{\{\max(S_{t_0}, \ldots, S_{t_n}) < B\}}
\]
5.3. Analysis Procedure

- simulate the spot with Monte Carlo
5.3. Analysis Procedure

- simulate the spot with Monte Carlo
- model the ECB-fixing $F_t$ by

$$F_t = S_t + \varphi, \quad \varphi \in \mathcal{N}(\mu, \sigma)$$
5.3. Analysis Procedure

- simulate the spot with Monte Carlo
- model the ECB-fixing $F_t$ by
  \[ F_t = S_t + \varphi, \quad \varphi \in \mathcal{N}(\mu, \sigma) \]
- $\mu$ and $\sigma$ estimated from historic data.
5.3. Analysis Procedure

- simulate the spot with Monte Carlo
- model the ECB-fixing $F_t$ by
  
  $F_t = S_t + \varphi, \quad \varphi \in \mathcal{N}(\mu, \sigma)$

- $\mu$ and $\sigma$ estimated from historic data.
- difference of fixing and traded spot = normally distributed random variable.
5.3.1. Estimates

Estimated values for mean and standard-deviations of the quantity Spot - ECB-fixing from historic time series\(^a\).

<table>
<thead>
<tr>
<th>Currency pair</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Time horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR - USD</td>
<td>-3.125E-6</td>
<td>0.0001264</td>
<td>23.6 - 08.8.04</td>
</tr>
<tr>
<td>USD - JPY</td>
<td>-4.883E-3</td>
<td>0.0134583</td>
<td>22.6 - 26.8.04</td>
</tr>
<tr>
<td>USD - CHF</td>
<td>-1.424E-5</td>
<td>0.0001677</td>
<td>11.5 - 26.8.04</td>
</tr>
<tr>
<td>EUR - GBP</td>
<td>-1.330E-5</td>
<td>0.00009017</td>
<td>04.5 - 26.8.04</td>
</tr>
</tbody>
</table>

For USD-JPY take EUR-JPY / EUR-USD etc.

\(^a\)Data provided by Commerzbank
5.4. Error Estimation

1. introduce a bid/offer-spread $\delta$ for the spot, which is of the size of 2 pips in the inter bank market.
5.4. Error Estimation

1. introduce a bid/offer-spread $\delta$ for the spot, which is of the size of 2 pips in the inter bank market.

2. evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold.
5.4. Error Estimation

1. introduce a bid/offer-spread $\delta$ for the spot, which is of the size of 2 pips in the inter bank market.

2. evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold.

3. compute for each path the error encountered due the fixing being different from the spot.
5.4. Error Estimation

1. introduce a bid/offer-spread $\delta$ for the spot, which is of the size of 2 pips in the inter bank market.

2. evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold.

3. compute for each path the error encountered due the fixing being different from the spot.

4. average over all paths.
5.4. Error Estimation

1. introduce a bid/offer-spread $\delta$ for the spot, which is of the size of 2 pips in the inter bank market.

2. evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold.

3. compute for each path the error encountered due the fixing being different from the spot.

4. average over all paths.

5. do this for various currency pairs, parameter scenarios, varying the rates, volatilities, maturities, barriers and strikes.
5.4. Error Estimation

1. Introduce a bid/offer-spread $\delta$ for the spot, which is of the size of 2 pips in the inter bank market.

2. Evaluate the payoffs for barrier options for each path and run the simulations with the appropriate delta hedge quantities to hold.

3. Compute for each path the error encountered due the fixing being different from the spot.

4. Average over all paths.

5. Do this for various currency pairs, parameter scenarios, varying the rates, volatilities, maturities, barriers and strikes.

6. We expect a significant impact particularly for reverse knock-out barrier options due to the jump of the payoff and hence the large delta hedge quantity.
5.5. Hedging Error for the Discretely monitored up-and-out Call

Two sources of error
5.5. Hedging Error for the Discretely monitored up-and-out Call

Two sources of error
5.5. Hedging Error for the Discretely monitored up-and-out Call

Two sources of error

Let $\Delta(S_t)$ be the theoretical delta (negative near $B$).
5.5.1. Spot Below Fixing: $S_t < B$ and $F_t \geq B$
5.5.1. Spot Below Fixing: $S_t < B$ and $F_t \geq B$

Here we unwind our hedge with delay and encounter
5.5.1. Spot Below Fixing: $S_t < B$ and $F_t \geq B$

- here we unwind our hedge with delay and encounter

$$\text{P&L} = \Delta(S_t) \cdot (S_{t+\Delta T} - S_t),$$

- the seller has been short the underlying at time $t$ and must buy it in $t + \Delta T$ minutes to close out the hedge.
5.5.1. Spot Below Fixing: $S_t < B$ and $F_t \geq B$

- here we unwind our hedge with delay and encounter

$$P\&L = \Delta(S_t) \cdot (S_{t+\Delta T} - S_t),$$

- the seller has been short the underlying at time $t$ and must buy it in $t + \Delta T$ minutes to close out the hedge.

- he makes profit if the underlying is cheaper in $t + \Delta T$. 
5.5.2. Spot Above Fixing: \( S_t \geq B \) and \( F_t < B \)
5.5.2. Spot Above Fixing: $S_t \geq B$ and $F_t < B$

- here the seller closed out the hedge at time $t$, though she shouldn’t have done so
5.5.2. Spot Above Fixing: $S_t \geq B$ and $F_t < B$

- here the seller closed out the hedge at time $t$, though she shouldn’t have done so

- and in $t + \Delta T$ she needs to build a new hedge causing

$$P&L = \Delta(S_t) \cdot (S_t + \delta) - \Delta(S_{t+\Delta T}) \cdot S_{t+\Delta T}$$
5.5.3. Calculating the Delta-Hedge Quantity

• approximation by Per Hörfelt in [6]

• Assume the value of the spot is observed at times $iT/n$, $i = 0, \ldots, n$

• define
5.5.3. Calculating the Delta-Hedge Quantity

- approximation by Per Hörfelt in [6]
- Assume the value of the spot is observed at times \( iT/n, i = 0, \ldots, n \)
- define

\[
\theta_{\pm} \triangleq \frac{r_d - r_f \pm \sigma^2/2}{\sigma} \sqrt{T}
\]

\[
c \triangleq \frac{\ln(K/S_0)}{\sigma \sqrt{T}}
\]

\[
d \triangleq \frac{\ln(B/S_0)}{\sigma \sqrt{T}}
\]

\[
\beta \triangleq -\zeta(1/2)/\sqrt{(2\pi)} \approx 0.5826
\]
5.5.3. Calculating the Delta-Hedge Quantity

- approximation by Per Hörfelt in [6]
- Assume the value of the spot is observed at times $iT/n, i = 0, \ldots, n$
- define

$$\theta_{\pm} \triangleq \frac{r_d - r_f \pm \sigma^2/2}{\sigma} \sqrt{T}$$

$$c \triangleq \frac{\ln(K/S_0)}{\sigma \sqrt{T}}$$

$$d \triangleq \frac{\ln(B/S_0)}{\sigma \sqrt{T}}$$

$$\beta \triangleq -\zeta(1/2)/\sqrt{(2\pi)} \approx 0.5826$$

- $\zeta$: Riemann zeta function
• define
\[
F_+(a, b; \theta) \triangleq \mathcal{N}(a - \theta) - e^{2b\theta} \mathcal{N}(a - 2b - \theta)
\]
• define

\[ F_+(a, b; \theta) \triangleq \mathcal{N}(a - \theta) - e^{2b\theta} \mathcal{N}(a - 2b - \theta) \]

• obtain for the value of the discretely monitored up-and-out call

\[
V(S_0, 0) \approx S_0 e^{-r_f T} \left[ F_+(d, d + \beta/\sqrt{n}; \theta_+) - F_+(c, d + \beta/\sqrt{n}; \theta_+) \right] \\
- Ke^{-r_d T} \left[ F_+(d, d + \beta/\sqrt{n}; \theta_-) - F_+(c, d + \beta/\sqrt{n}; \theta_-) \right]
\]
• define

\[ F_+(a, b; \theta) \triangleq \mathcal{N}(a - \theta) - e^{2b\theta} \mathcal{N}(a - 2b - \theta) \]

• obtain for the value of the discretely monitored up-and-out call

\[
V(S_0, 0) \approx S_0 e^{-r_f T} \left[ F_+(d, d + \beta/\sqrt{n}; \theta_+) - F_+(c, d + \beta/\sqrt{n}; \theta_+) \right] \\
- K e^{-r_d T} \left[ F_+(d, d + \beta/\sqrt{n}; \theta_-) - F_+(c, d + \beta/\sqrt{n}; \theta_-) \right]
\]

• take a finite difference approach for the computation of the theoretical delta

\[
\Delta = V_S(S, t) \approx \frac{V(S + \epsilon, t) - V(S - \epsilon, t)}{2\epsilon}
\]
5.6. Analysis of EUR-USD

Spot 1.2100
Strike 1.1800
Trading days 250

domestic interest rate 2.17% (USD)
Foreign interest rate 2.27% (EUR)

Volatility 10.4%
Time to maturity 1 year

Notional 1,000,000 EUR

Consider a short position of a discretely monitored up-and-out Call
### 5.6.1. Distribution of Absolute Errors in USD

The figures are the number of occurrences out of 1 million

<table>
<thead>
<tr>
<th>Barrier 1.2500</th>
<th>&lt;1k$</th>
<th>&lt;2k$</th>
<th>&lt;3k$</th>
<th>&lt;39k$</th>
<th>&lt;40k$</th>
<th>&lt;41k$</th>
<th>&lt;42k$</th>
<th>&lt;43k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upside Error</td>
<td>951744</td>
<td>20</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Downside Error</td>
<td>48008</td>
<td>54</td>
<td>2</td>
<td>5</td>
<td>59</td>
<td>85</td>
<td>21</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Barrier 1.3000</th>
<th>&lt;1k$</th>
<th>&lt;2k$</th>
<th>&lt;3k$</th>
<th>&lt;89k$</th>
<th>&lt;90k$</th>
<th>&lt;91k$</th>
<th>&lt;92k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upside Error</td>
<td>974340</td>
<td>20</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Downside Error</td>
<td>25475</td>
<td>43</td>
<td>0</td>
<td>2</td>
<td>40</td>
<td>59</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Barrier 1.4100</th>
<th>&lt;1k$</th>
<th>&lt;2k$</th>
<th>&lt;3k$</th>
<th>&lt;199k$</th>
<th>&lt;200k$</th>
<th>&lt;201k$</th>
<th>&lt;202k$</th>
<th>&lt;203k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upside Error</td>
<td>994854</td>
<td>78</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Downside Error</td>
<td>4825</td>
<td>194</td>
<td>3</td>
<td>1</td>
<td>19</td>
<td>17</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>
5.6.2. Additional Hedge Cost

Hedge error with 99.9% - confidence interval

barrier

error

error in USD
confidence band
5.6.3. Probability of a Miss-Hedge

![Graph of Probability of mishedging](image)

- Probability of mishedging: 0.00% to 14.00%
- Barrier: 1.22 to 1.46
- Probability: 0.00% to 14.00%
5.6.4. Hedging Error / TV

Rel. hedge error with 99.9% bands
Rel. hedge error with 99.9% bands
5.6.5. Maximum Losses

Extremal P & L for the short position

-300000 -250000 -200000 -150000 -100000 -50000 0 50000

P & L

1.22 1.25 1.28 1.31 1.34 1.37 1.4 1.43 1.46

max downside error

max. upside error
5.6.6. Maximum Losses / TV

![Graph showing relative P & L](image)

- Relative P & L
- Max downside error
- Max upside error

<table>
<thead>
<tr>
<th>Barrier</th>
<th>P &amp; L in USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.24</td>
<td>-45000%</td>
</tr>
<tr>
<td>1.26</td>
<td>-40000%</td>
</tr>
<tr>
<td>1.28</td>
<td>-35000%</td>
</tr>
<tr>
<td>1.3</td>
<td>-30000%</td>
</tr>
<tr>
<td>1.32</td>
<td>-25000%</td>
</tr>
<tr>
<td>1.34</td>
<td>-20000%</td>
</tr>
<tr>
<td>1.36</td>
<td>-15000%</td>
</tr>
<tr>
<td>1.38</td>
<td>-10000%</td>
</tr>
<tr>
<td>1.4</td>
<td>-5000%</td>
</tr>
<tr>
<td>1.42</td>
<td>0%</td>
</tr>
<tr>
<td>1.44</td>
<td>5000%</td>
</tr>
<tr>
<td>1.46</td>
<td>10000%</td>
</tr>
</tbody>
</table>
5.7. Summary

- other currency pairs are similar.
5.7. Summary

- other currency pairs are similar.
- average loss is comparatively small for liquid currency pairs.
5.7. Summary

- other currency pairs are similar.
- average loss is comparatively small for liquid currency pairs.
- maximum loss can be very large with small probability.
5.7. Summary

- other currency pairs are similar.
- average loss is comparatively small for liquid currency pairs.
- maximum loss can be very large with small probability.
- sufficient to charge a maximum of 0.1% of the TV to cover the potential average loss.
5.7. Summary

- other currency pairs are similar.
- average loss is comparatively small for liquid currency pairs.
- maximum loss can be very large with small probability.
- sufficient to charge a maximum of 0.1% of the TV to cover the potential *average* loss.
- traders take extra premium of 10 basis points per unit of the notional of the underlying.
5.7. Summary

- other currency pairs are similar.
- average loss is comparatively small for liquid currency pairs.
- maximum loss can be very large with small probability.
- sufficient to charge a maximum of 0.1% of the TV to cover the potential *average* loss.
- traders take extra premium of 10 basis points per unit of the notional of the underlying.
- relative errors are so small that it seems reasonable not to pursue any further investigation with other models beyond Black-Scholes.
6. Contact Information

Uwe Wystup
MathFinance AG
Schießhohl 19
65529 Waldems
Germany
Phone +49-700-MATHFINANCE

More papers are available at
http://www.mathfinance.com/wystup/papers.php

These slides are available at
References


